1942

Design of gambrel barn roofs

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DESIGN OF GAMBREL BARN ROOFS

by

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Value of farm buildings

Although the investment in a single farm building is relatively low, the total investment represents an enormous capital. Farm buildings in the United States are valued at over ten billion dollars (26). Iowa ranks first among the individual states with an investment of over 794 million dollars. Any investment of such a magnitude demands considerable study in order to lower the rate of depreciation. Properly designed farm buildings with correct methods of construction applied would lower the rate of depreciation.

Giese (17) states, "The American farmer maintains a huge investment in farm buildings. Although not spectacular in nature, they exert a very definite influence upon agricultural welfare by influencing the costs of production, the preservation of farm products, and the standards of rural living. These factors are of sufficient importance to justify the application of most advanced thought to the matter of farm structures design. Nevertheless, workers responsible for the planning of farm buildings have been handicapped by lack of design data based upon adequate research."

Wind damage to farm buildings

Many of our farm buildings have been designed with little knowledge as to the effect of the wind on the building. Very few designers have taken into account the large negative pressures on the leeward side of buildings which tend to lift the roof off of the building. From observation of wind damage (20), the lifting effect of wind on the roofs of wood structures is responsible for more failures of buildings to withstand wind storms than any other one cause.

According to Giese (16), a study of losses of farm buildings due to wind storms in Iowa, indicates that Iowa farmers pay a large annual tribute to poor construction methods. Experiences as reported by Wooley show further that poor construction is not peculiar to Iowa but may be observed in other states.

The average annual monetary loss, in farm buildings in the State of Iowa during the period 1930-1938, was $323,308 as paid by the Iowa Mutual Tornado Insurance Company. Such a loss is a great economic waste to the rural people and an effort to eliminate this loss is certainly justified.

The house, which comprises over half of the farm building investment, has suffered least of all when considering the size of investment. Also, the percentage of the dwellings damaged has been considerably smaller than for other farm buildings as shown in Figure 1. This fact alone indicates that it is possible to build to withstand windstorms. However, the added cost which would be necessary in order to make a barn completely wind proof may not be justified. Still, there is a reason to believe that through proper methods of construction it is possible to eliminate a large per cent of the wind damage to farm buildings.

Wood as a building material

Wood, as a farm building material, still holds supremacy over other kinds of structural materials. This has been partially due to cheapness of construction with wood. Strength and stiffness combined with lightness are other qualities which have kept wood first. Weight for weight, wood is stronger than steel. It can also be grown as a crop without depleting natural resources and is easily fabricated. The ease of fabrication may actually be a handicap if it is assumed that anyone can fabricate wood, as it is evident that poorly constructed buildings may be the result of the use of unskilled labor.

Betts (3) makes the following statement about the design of woodframe buildings:

"In the designing of the actual structure, that is, the putting together of the various materials, common local practice is the usual guide. Practice varies with locality, Precedent is a potent factor. Because of ignorance of

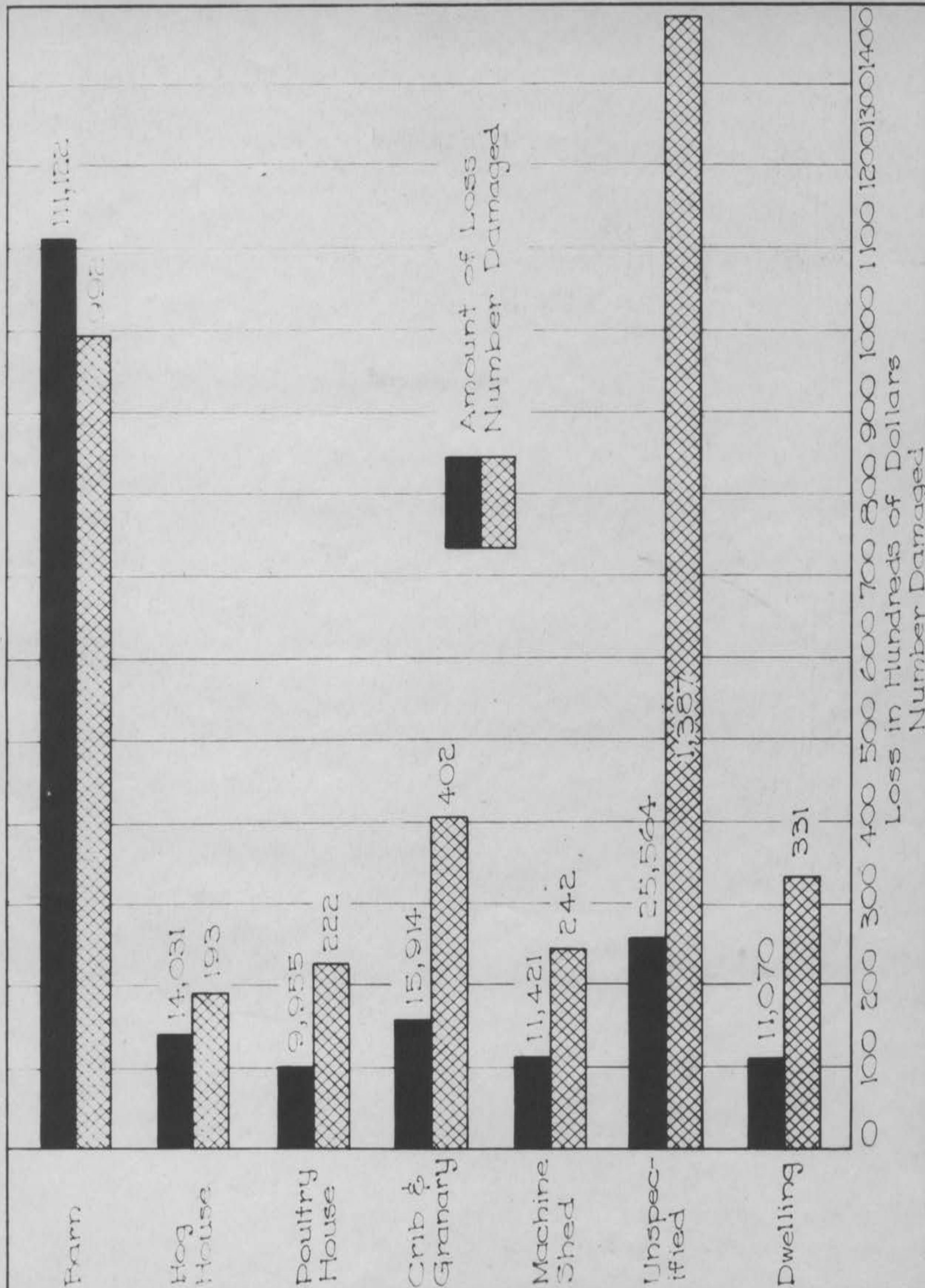


Fig. 1 Total Damage To Farm Buildings By Wind
Average 1930-1933 (Iowa)

requirements, strength of material or engineering principles, the designers of some of our existing buildings built safely by employing considerable more material than may have been necessary -- others, for the same reasons, produced buildings that have failed structurally."

Inadequacy of present gambrel barn roof design

Numerous designs of gambrel barn roofs are now available to farmers for use in construction of barns. However, the vast majority of these designs are not based on the fundamental principles of mechanics. As a result, they do not render the service that better designs would give. Inadequately braced roofs and roofs which have bracing placed in the wrong position are much more likely to fail in a windstorm than a properly braced roof. In many of our present gambrel barn roof designs, sagging along the ridge is particularly noticeable. Others do not facilitate the use of hay handling equipment. Poor proportioning is another defect which is quite noticeable. A barn that is not properly proportioned detracts from the appearance of the farmstead.

Figure 2 shows a poor design of a gambrel barn roof and a good design of a gambrel barn roof. In the poor design the change in the angle of inclination with the horizontal from the lower rafter to the upper rafter is so small that practically no advantage was obtained by using this type of

Fig. 2. A Poor Design and a Good Design of a Gambrel Roof

barn. It is also much more likely to sag along the ridge than the good design. The good design also furnishes a larger mow space in proportion to the square feet of roof area.

Object of the Study

The specific objectives of this study are:

1. To combine and correlate the work of previous investigators;

2. To correct the weakness of the conventional designs now in use;

3. To develop rafter plans that are of value to the farm builder.

The general objective of this study is to develop better methods of construction for the utilization of wood in the design of gambrel barn roofs.

HISTORICAL

The Project

Project setup

Project 563, "The Utilization of Lumber in Farm Building Construction," was initiated into the Iowa Agricultural Experiment Station in 1937. The project is sponsored by the Weyerhaeuser Sales Company and has as its objective the improvement of farm buildings through the use of wood in improved methods of construction. The Weyerhaeuser Sales Company believed that many of the farm building failures were due to improper methods used in construction. It was also believed that many of the buildings that were in good shape were over-designed and that through the study of the physical and mechanical properties of wood over-designing could be eliminated.

Previous investigations

Previous work on this project dealt mainly with the improvement of the corn crib and granary. After considerable study of the various requirements of grain storage buildings, Richardson (24) began the project with a study of the structural improvement of the corn crib and granary. With the object, "to use materials more efficiently," in mind, Richardson developed a self-supporting grain bin partition over the central driveway of the building.

Crawford (7), the second man to work on this project, continued the work of Richardson by devoting his time to the structural analysis and design of a corn crib. He attacked the problem by designing with reference to the arrangment and functions of the various parts of the structure. Crawford also studied and tested several types of bracing and joints that have possibilities of use in construction of other farm buildings.

Rice (23), the third man to work on this project, devoted his year's work to the analysis and design of a rigid frame gambrel barn roof. In his work he compared the conventional two rafter gambrel barn roof with the three rafter gambrel barn roof. The value of using glued and nailed gusset plates at joints was studied.

Lowery (19), the fourth man to work on this project, combined and correlated the work of previous investigators on the study of the granary and corn crib. With a more efficient use of materials and improved methods of construction in mind, Lowery developed a set of plans for corn cribs and grain storage structures. Plans for a triple corn crib were also developed.

Until this investigation, further work had not been done on this project. However, investigations and work carried out by other projects in the same institution have helped considerably in developing better methods of construction for farm buildings.

History of Barn Framing

The trussed rafter

The light woodframe construction of the modern barn has been developed through a gradual change of design from the massively framed timber barns of a half century ago. The Clyde Roof Truss and the Shawver Roof Truss (4) are roof trusses that supplanted the heavy type of framing in early changes of design. These trusses were strong and made a rigid structure. For several years this method of construction was popular, but it has gradually given way to a still lighter method of framing.

The braced rafter

The braced rafter roof has become one of more popular types of framing in the modern barn. This is due to several reasons. The braced rafter is a light, rigid method of framing. Each rafter is a complete unit and can be easily constructed and raised. This type of construction is economical and fits well into the program of utilizing wood as a farm building material.

Gothic arch

Although barns with Gothic arch roofs were built as early as 1885 in Isabella County, Michigan (12), it has been only in the past few years that they have become popular. The first curved rafters were made of 2"x2" members with the top edge sawed to the desired curvature. Others were made by sawing short 1" boards, 8" or 10" wide and 3' or 4' long to some desired curvature and nailing together with staggered joints. Later the curved rafters were made by bending four or five plies of 1"x4" material into the arc of a circle. Each lamination was securely nailed to the other laminations. The modern design (21) consists of five to nine laminations of 1"x2" material bent into the arc of a circle. These laminations are held in place by water resistant glue and nails. The Gothic arch is becoming more popular due to its pleasing appearance and the clear mow space it provides.

Summary of Previous Investigations

As one of the main objects of this study is to combine and correlate the results found by previous investigators, a summary will be made of these investigations.

Schweers (25), in an investigation of farm building losses due to wind, found from observation that many losses were due to poor design and construction. The plate joint used in construction was particularly inadequate. In tests on some of the conventional rafter designs no advantage of using a long brace from the stud to the rafter was found. A short brace fitting up close to the plate proved to be a better brace. The greatest strength was secured by lapping the stud and rafter. All the joints used in the tests were nailed.

Arnold (1), in a study of design of roof trusses to resist wind loads, found it essential to consider the outward pressure on the walls and roof. In wind load tests on the Clyde Roof Truss, the plate joint was found to be weak. With the plate joint reinforced with iron straps, the truss provided a factor of safety of 2 in wind velocity and 4 in wind load. In studying the effect of roof shape on the appearance of the building and farmstead, the gambrel roof barn was considered to harmonize fairly well with the other farm buildings. It was also pointed out that it was possible to secure as large a mow space as desirable by increasing the distance from the mow floor to the plate end and by making the barn longer.

Pickard (22) made a study of the braced rafter roof. As the joints used in rafter construction seemed to be weak, and in many cases the cause of failures, the first part of the investigation was given to the study of joints. The purpose of this study was to determine the most efficient method of connecting the members of the braced rafter roof. Tests were made of glued, nailed, bolted, and timber connector joints. The glued joints gave the highest ultimate strength when the load was applied parallel to the grain of the wood. The results obtained in the tests of the bolted and timber connector joints indicated a strong joint. The nailed joint was the weakest of all joints tested.

By a stress analysis it was determined that under ordinary wind loads the greatest moment on the rafter was at a point approximately two-thirds of the distance from the plate line to the upper rafter. With this in mind, Pickard developed a new rafter that was much stronger than the conventional type. The new rafter failed only after a load equivalent to that produced by a 245 mile per hour wind was applied. The conventional type of rafter failed under a load produced by a 175 mile per hour wind. The use of glue was also found to increase the strength of the rafters.

Rice (23) made a study of the analysis and design of the rigid frame gambrel barn roof. In this study further investigation was made in regard to the value of the use of glue in rafter construction. Tests using gusset plates for splicing and joining rafters showed that the spliced member approached the strength of the continuous member when the plate was glued and nailed. The three rafter gambrel barn roof was found to have considerable possibilities in this study.

According to stress analysis and tests, stresses are greatly reduced in rafters by using rigid frame construction. Using the same rafter design for tests on rigid frames and three hinged arches, the rigid frame failed under a load produced by a 120 mile per hour wind, while the three hinged arch failed when subjected to a 90 mile per hour wind load.

In both Pickard's and Rice's studies the advantages of using stable shapes for the design of gambrel barn roofs were indicated.

Review of Literature

Barn framing requirements

Service requirements. In designing a barn there are a number of functional requirements that must be considered. These framing requirements must be provided in all types of barns. They are listed as follows:

1. It must provide adequate space and shelter for animals.

2. It must provide adequate mow space for the storage of feed.

3. It must provide width and height that facilitate the

use of hay handling equipment without lowering the track too far from ridge.

Structural requirements. Strength, stability and rigidity are the structural demands of barn framing. The barn must have sufficient strength to sustain all loads that it will be forced to carry. Wind loads may cause excessive stresses, unless it has sufficient strength to meet these loads. The gusty nature of the wind causes large stresses in unrigid structures. Joints are also more likely to loosen if the requirement of rigidity is not met. A few extra dollars spent in securing rigidity may add many years of service life to the building.

A barn roof shape that is stable under all dead loads is desirable. The stresses caused by dead loads are not extremely large but over a long period of time deflection may be the result. With the resultant of the dead loads acting through the joints, the structure is less likely to lose its shape.

Economic requirements. The economy of a structure is a requirement to which careful consideration should be given. In securing an economical building it is necessary, (1) to select a barn size that meets the requirements of the farm, (2) to use standard dimension lumber, (3) to use local building material, and (4) to secure economy of labor as far as feasible.

The economy of a building depends considerably on the first cost and the subsequent charges for repair and depreciation. The use of shoddy material and poor methods of construction, to secure a low first cost, may cause an undue cost of upkeep that will more than overbalance the economy secured in construction. The roof material must be durable and must provide protection for the supporting members.

Cartwright (5) says, "Once built, it is desired that a building shall last as long as possible, and some care must be taken to that end. It is easily possible to build a frame structure which, if well maintained, will have a very long life of usefulness. There are, as lumbermen are prone to point out, any number of frame dwellings built in colonial days now two and three hundred years old, which are still in good condition and occupied, in some cases by the descendents of those who built them. The secret for the most part is preservation from moisture, which means, for lumber, preservation from decay. Lumber exposed to the weather also will wear away slowly through disintegration of the cellular structure where no decay occurs."

Aesthetic requirements. The appearance of the barn has a definite influence upon the rural people. Beauty on the farm is essential to the maintenance of morale and high standards of living. The barn is an individual unit, and harmony with the other buildings on the farm is not essential. However, the barn should be well proportioned and pleasing to the eye, giving the impression of permanence and stability. The barn should be designed to meet the requirements governed by stock and feed storage space. A large farm does not always require a large barn.

Selection of barn sizes

An investigation by Barre (2) found 32, 34, and 36 feet to be the common widths of the majority of the barns in use. Figure 3 shows the results of the investigation of barn widths and recommended barn plans. In more recent years there has been a demand for a 40 foot barn.

The width of the lower structure is determined mainly by the space required for housing the animals. A more economical lower space is secured by using the 40 foot width of barn and shortening the length of the barn. The height of the roof is usually determined by the storage capacity required. Another factor that may influence the height of barn to be used is the wind load on the structure. An increase in height of the barn causes an increase in wind load. Changes in methods of storing feed and housing animals tend to change the dimensions to be used.

Design of the barn

With the desirable width of barns in mind, careful

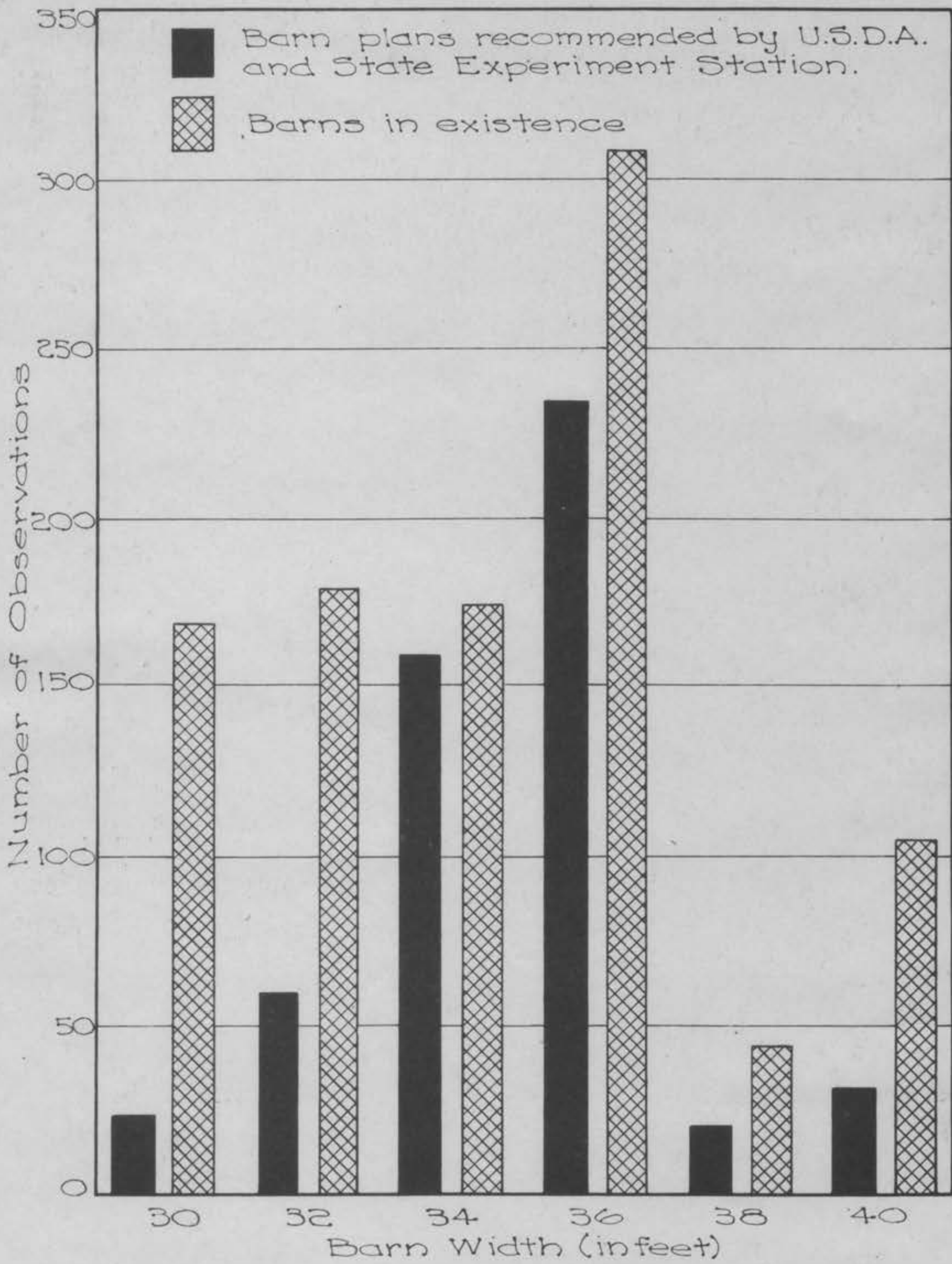


Fig. 3 - Observation of Barns and Barn Widths in Iowa.

consideration should be given to the design. The main stresses to which the barn will be subjected are caused by dead load and wind load. Until the last few years little or no effort was made to take into account the wind load on the barn. Few facts were known about the properties of wood and its action under various loads. Local practice has been the guide. Material in the vast majority of farm buildings has been placed according to the estimates of local carpenters.

In designing a barn it is necessary to make a mathematical analysis of the stresses caused by the various loads. In the vast majority of the designs presented at the present, no stress analysis has been made. A good many designs are offered that are supposed to withstand loads of high wind velocities. Some of these designs will stand severe wind loads as they are securely braced with numerous braces to insure their strength. The purpose of the stress analysis is to determine the critical points or places subjected to large stresses in the structure. With a definite knowledge of the severely stressed points, it is possible to locate the braces in the correct place. It is also possible, in a poor design, for a weak member to deflect, causing undue stress in another member, which may result in failure at this latter point. As a result it is possible that some members in the structure may be overbraced while others lack sufficient bracing.

The method of fabricating joints is another important point that must be considered in design of the barn. Joints have always been a critical factor in wood construction.

Timber fasteners

The joint in wood construction has long been recognized as a very weak link. The strength of woodframe structures has been limited to a large extent by the inability to furnish a strong connection from one member to another. Recent developments have made it possible to secure stronger connections than were once available. In many cases, however, these stronger fasteners are not being used. A study of fasteners presents many interesting facts.

Nails. Nails, due to their easiness of use, will never be completely replaced as a fastener in wood structures. In some places nails give many useful advantages. For fast construction they may rank first as a timber connector, but when strength is considered, they rank very low.

According to tests by Pickard (22), nails offer little shear resistance. Immediately upon the application of a load, the nailed joint begins to deflect or slip. This is partially due to the small bearing area of the nails which crushes the fibers of wood under a small load. The ultimate strength of the nailed joint was also smaller than the ultimate strength of joints using other types of fasteners. Other fasteners used were glue, bolts, and timber connectors.

Giese (14) writes, "Particular care must be taken at the joints. Nails, although easy to use because of the ease of driving, are comparatively ineffective. It is physically impossible to drive a sufficient number of nails into the end of a structural member to make the joint comparable in strength to the timber as a beam or as a brace. Where splitting occurs, what little strength the nail joint had disappears. A few well placed bolts will return very satisfactory dividends, but still better results can be obtained by the use of timber connectors."

Few of the recommended designs specify any method of connecting members other than by nails.

Bolts. The bolted timber joint is considerably stronger than the nailed joint. This is partially due to the increase in the bearing area between the wood and the fastener. The deflection and slip is considerably lower and the ultimate strength is higher for bolts than for nails. Bolts are more expensive than nails and they are harder to use, as a hole must first be made before the bolt can be placed in position. Bolts are convenient fasteners to use in temporary construction as they can be removed with little trouble. The use of bolts in permanent construction of farm buildings may be warranted if they increase the strength of the joints, which in turn increases the life of the building.

Timber connectors. One of the more recent timber fasteners that has lately been inducted into farm building construction is the modern connector. The modern connector was developed in Europe soon after the beginning of the World War. Prior to that time casein glue had been used extensively, but trade barriers prevented the importation of the casein from which the glue was made; hence, the timber connector was developed to substitute for glue.

Modern timber connectors in general consist of metal rings or plates and disks that, embedded partly in each of adjacent members, transmit the load from one member to the other. Timber connectors have proven their value by use in the construction of timber trusses. Connectors make it possible to use a much higher value for the allowable working stresses.

Cartwright (6), in discussing the possibilities for the use of timber connectors in farm buildings, made the following statement: "The importance of timber connectors in timber framing may be judged from the fact that they increase load bearing capacity in structures such as mentioned above from 50 to 100 per cent. Design, with timber as a material, has in the past been limited by the strength of connections. Bolted connections were capable of developing on the average only from 50 to 60 per cent of the allowable working stresses in the members. It is possible in most cases with timber connectors, to realize substantially the full allowable working stresses of material and the result is that much greater load capacity can be developed with the same amount of material, or, conversely the same loads can be provided for with reduced footage."

In tests carried out by Pickard (22), on joints and braced rafters constructed with timber connectors, good results were obtained. Indications were that timber connectors had a definite place in timber construction of all kinds. The timber connector may be of special aid to the utilization of lumber in farm buildings.

Glued joints. In Europe, as mentioned, timber connectors largely supplanted glue as a timber fastener in the World War. In the War of Survival it may be reversed in this country. The scarcity of steel for defense projects and priorities on steel may limit its use for timber connectors and nails and open the way for the use of glue, which is exceptionally good for connecting wood members.

Although glue has been used in joining wood members for many years, its use has only recently been brought into the field of farm structures. At the present glue is used only in limited quantities, as its durability is sometimes questioned. However, casein glue and other types of water resistant glues have been found to be almost as durable as the wood itself.

In order to determine the durability of glue, Wilson (28), Senior Engineer of the Forest Products Laboratory, made an inspection of glued laminated construction in Europe. The buildings had been constructed from 20 to 35 years. In practically every case the glued joints were in good condition. After a careful study of the use of casein glue in laminated construction, the following statement was made:

"From experience to date it seems safe to assume that casein-glued laminated construction will last as long as solid wooden members of any but the more durable species or preservatively treated material."

With the durability of glue equal to that of the average species of wood used in farm building construction, it is possible to use it to improve the methods of fabricating joints. According to Giese (16), the use of glue in farm building construction makes possible the use of rigid frame construction. By the use of gusset plates and glue, covering as large an area as needed at joints, it is possible to secure a strong joint that is able to resist moment.

Tests carried out by Martin (20) show the shear value of glue to be between 300 p.s.i. and 400 p.s.i. In most of the pieces tested, not more than one-half to two-thirds of the material was actually connected with glue. 500 p.s.i. to 550 p.s.i. seemed to be a conservative estimate of the ultimate shear stress of glue when the full surface is properly glued. In each test glue developed over twice the allowable shear stress for wood, which is approximately 150 p.s.i. Glue is capable of developing five times the allowable shear stress for wood. In most of the tests glued joints that had been subjected to eight years of service conditions were used. For this period of time the quality of the glue was unaffected.

According to Pickard (22) the ultimate strength of the glued joint perpendicular to the grain is one-third of the strength when the load is applied parallel to the grain. This is due to the nature of these joint failures. The fibers of the center timber can be torn apart perpendicular to their axes much more easily than in the line of the axes. Even with this significant difference the low strength glued joints were as strong as the joints fastened by any other method.

The Forest Products Laboratory (27) has investigated the use of nails for pressing or clamping the glued members while the glue is setting. In the tests common 8d nails were used. One nail was provided for each 8-1/4 square inches of area for the glued joint. The shear tests showed the glued joints, with pressure furnished by nails, to have approximately two-thirds of the ultimate shear value of the joints connected with high pressures. However, in these tests the lowest shear stress was 488 p.s.i., while other stresses ranged as high as 1,140 p.s.i.

The results of this study indicate that glue may be of considerable value for use in farm building construction. Glue used in combination with nails will add greatly to the strength and rigidity of woodframe structures.

Wind pressure investigation

The greatest stresses to which a barn is subjected are those resulting from wind loads. Correct evaluation of these loads will help to make a successful design. It is possible through tests in wind tunnels to determine approximately the wind loads to consider on a structure. However, most of the wind loads that have been used in practice are based upon theoretical computations. Most of these theories disregard the shape of the structure and take into account only the impact pressure of the wind against the windward surfaces.

Methods of calculating wind loads. Many formulae have been developed from time to time by various people in an effort to show the relation between wind velocity and the pressure exerted by the wind on objects. The five most widely used formulae in determining wind loads are (1) Newton's, (2) Rankine's, (3) Hutton's, (4) Duchemin's, and (5) Smeaton's. All of these formulae differ to some extent and none of them take into consideration the negative pressure on the leeward side.

For calculating wind pressures on objects perpendicular to the wind, the formula $p = kv^2$, has been used considerably. p is the pressure in pounds per square foot; v is the velocity of wind in miles per hour; and k is a constant. The determination of the correct value of k has given considerable trouble.

Newton and Rankine, by purely theoretical calculations and disregarding the suction created on the leeward side of any objects, recommended values for k of .0027 and .0054, respectively. This fact illustrates the reason for so much confusion. In deriving the value of k, the shape of the object was not taken into consideration, hence it has no true value for general application in determining wind pressures.

The only recourse left in the determination of wind loads is experimentation. By experiments it is possible to determine fairly close values for wind pressures.

Experimental investigation. Wind pressures depend not only upon the wind speed and direction, but also upon the shape or form of the structure against which the wind is blowing. Hence, there is no fixed pressure corresponding to a certain wind speed that is applicable to all shapes of structures. Almost the only way to determine wind pressures on a structure is by actual measurements of these pressures, either on the structure itself or on a model of the structure. The variation of wind velocity and direction under natural conditions cannot be controlled, therefore it becomes necessary to resort to the use of the wind tunnel.

By the use of the wind tunnel the wind velocity and direction are easily controlled. However, there are several factors in using the wind tunnel that must be taken into consideration. The disadvantages of this method are: (1) that actual conditions of the wind cannot be reproduced, (2) that the fine detail of the actual building cannot be reproduced on the model, and (3) that the pressure on the full-size building may vary somewhat from that at the corresponding position on the model due to "scale-effect." In regard to the errors resulting in an experiment on scale models, Dryden and Hill (9) made the following statement:

"It is the judgment of the authors that the results obtained on a model can be applied directly to full-scale structures without causing errors of any consequence."

In considering the effect of wind on any structure, there are two important questions to be considered. They are:

1. What are the maximum loads caused by wind and how often do they occur?

2. What are the stresses in various members of the structure due to these loads?

These questions must be studied in the order mentioned. It is practically impossible to determine the wind stresses without first determining the wind load.

Dryden and Hill (8), in their investigation, used a method of calculating wind loads that took into account the reduced pressure on the leeward side.

When an air stream is blowing against an object, a pressure p is produced. The pressure p can be considered to consist of ps - pw, where ps is the static pressure and pw is that pressure caused by the object in the air stream. If there is no wind p = ps and pw is equal to zero. Wind pressure may be either positive, negative or zero. In a structure such as a barn, the pressure inside can be assumed to be equal to the static pressure. The resulting unit pressure is therefore equal to the difference in pressure on the opposite surfaces of the wall or roof. The wind pressure pw may then be expressed by the equation $\frac{pw}{q} = \frac{f\,(vLd)}{u}$ where q is the velocity pressure, d the air density, v the wind speed, u the viscosity of the air, and L a linear dimension indicating the scale. For bodies without curved surfaces and having sharp corners the expression $\frac{pw}{q}$ is practically independent of wind speed and size of object. Therefore $\frac{pw}{k} = K$ or $pw = Kq$ where $K = \frac{f\,(vLd)}{u}$, and depends only upon the location of the point on the structure. If the value of K is found for any point on a model, that value will apply to any size of the model at any wind speed.

The velocity pressure of the wind may be represented by the formula $\frac{wv^2}{2g}$, which is applicable to any moving body. For air at a temperature of 15°C. and a barometric pressure of

760 mm. Hg, the weight per cubic foot is .07651 lbs.

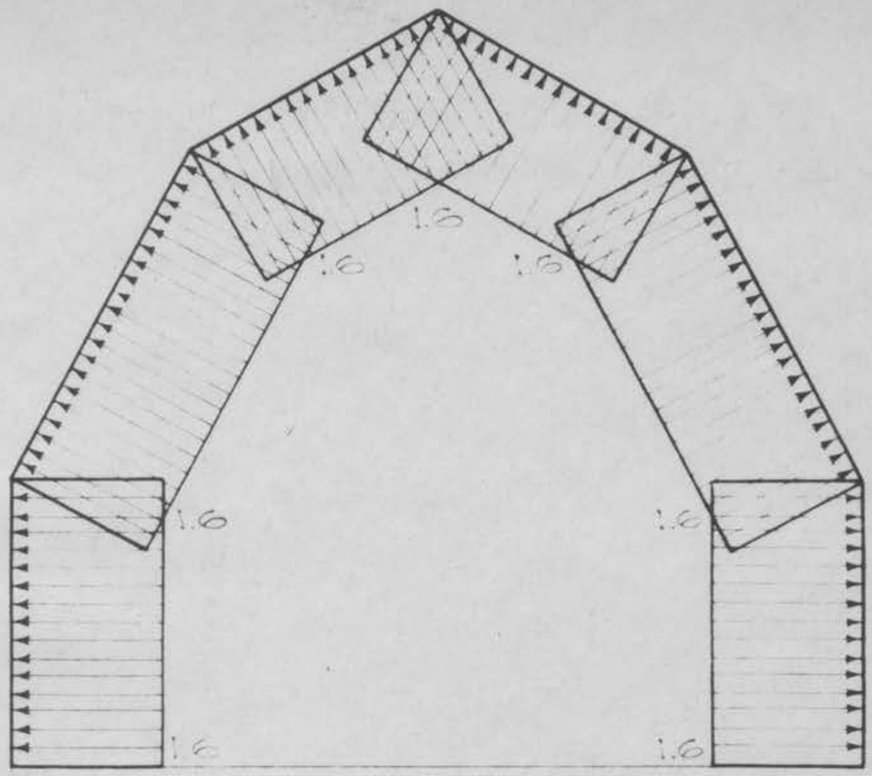
$$p = \frac{.07651v^2}{2 \times 32.2} = 0.001189v^2$$

If v denotes the velocity of wind in miles per hour,

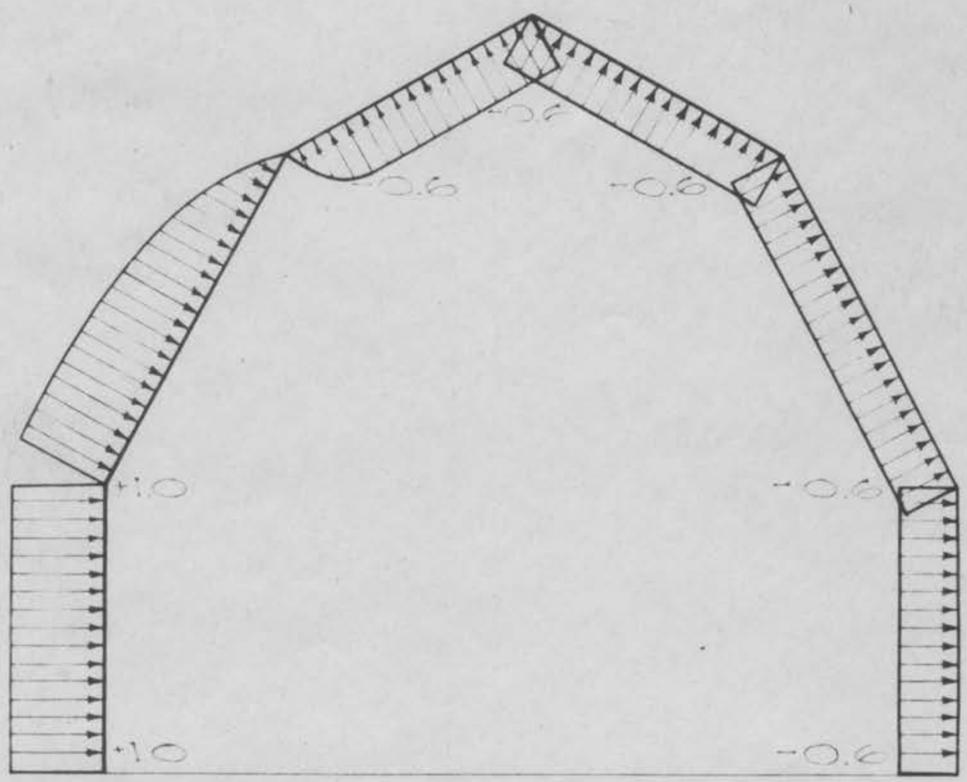
$$p = 0.001189 \left(v + \frac{22}{15}\right)^2$$

This formula was used in calculating the wind loads on the barn roof in this investigation.

The wind pressure distribution diagrams used in this investigation are shown in Figures 4 and 5. The diagrams for the wind pressures on the two gambrel barn roof were designed by Pickard (22). The diagrams for the wind pressure on the three rafter gambrel barn roof were designed by Rice (23). Each of these diagrams was approved by Dr. Hugh L. Dryden of the United States Bureau of Standards. They are also similar to wind distribution diagrams as found by Fenton and Otis (10).



WIND 90° TO END



WIND 90° TO SIDE

Fig. 4. Wind Pressure Distribution Diagrams.
2 rafter gambrel barn roof

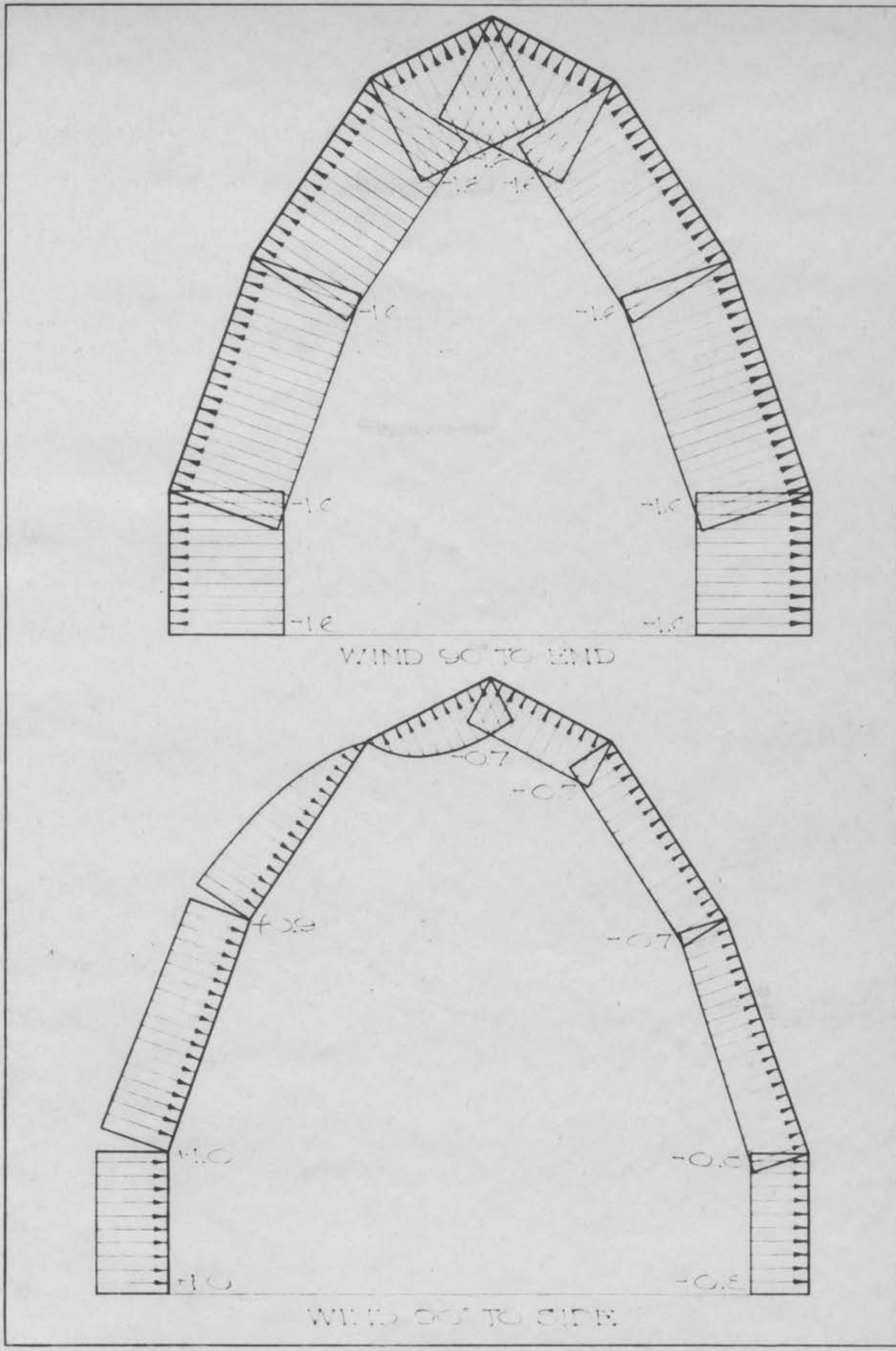


Fig. 5 - Wind Pressure Distribution Diagrams
3 rafter gambrel barn roof

THE INVESTIGATION

Justification

The design of gambrel barn roofs presents many problems. Some of these problems have been studied in previous investigations and have been solved in many cases. However, there are a number of questions which are still unanswered or only partially solved. An investigation of the unsolved problems and the correlation of previous studies are particularly important in designing gambrel barn roofs.

In the design of gambrel barn roofs it is impossible to determine accurately all the forces that must be resisted by each member. This is true for two reasons: (1) It is impossible to determine the loads to which the barn will be subjected; (2) It is impossible to determine how the different stresses caused by various loads will be distributed through the joints to the different members. The strength of joints varies with the different kinds of wood and the method of fastening or joining the members. The efficiency and value of braces depends upon the position, whether in compression or tension, and the method used in connection. Each kind of wood is also a material with variable qualities which further complicates its use.

At the present, there are no uniform gambrel barn roof designs. In the Midwest Farm Building Plan Service there are nine different methods of bracing gambrel barn roofs. An inspection of a number of gambrel barn roofs in Iowa revealed that practically every carpenter uses a different method of bracing the rafters. It is quite evident that no one knows what is the best design to use. In some cases the Clyde Roof Truss and the Shawver Roof Truss are still relied upon in the construction of gambrel barn roofs. The use of these trusses complicates construction, as they are extremely difficult to place in position. The purlins between the trusses tend to sag, causing a wavy appearance of the barn roof. The elimination of the poor designs and the determination of a good, uniform design will be of invaluable aid to the farmer.

Until this study the investigations of gambrel barn roof designs had been limited to an analysis of one or two barn roofs forone width barn. Obviously, it is impossible to meet the requirements of the various types of farms with one standard barn width and roof. An analysis and design of several of the more common widths of barns, with two or three different sizes of roofs, will enable the farmer to choose a barn that will meet the requirements of the particular farm in mind. He will also be able to secure a gambrel barn roof that is far more structurally sound than the present gambrel barn roof designs.

This investigation includes analyses and designs of both the conventional two rafter and three rafter gambrel barn roofs. Each of these types of roofs has certain advantages over the other type, depending upon the particular situation.

Specific Objectives

The specific objectives of this investigation are:

1. To determine stable shapes for the two and three rafter gambrel barn roofs for the 32', 34', 36' and 40' widths of barns

2. To compare stable shape gambrel barn roofs with recommended pitches

3. To determine bending moments and reactions resulting from dead loads

4. To determine bending moments and reactions resulting from combined dead and wind loads

5. To improve present methods of bracing and fastening gambrel barn roofs

6. To determine the most efficient rafter design possible

7. To work out details of construction for the two and three rafter gambrel barn roof designs

Procedure

Using the data obtained from previous investigations, a definite procedure was outlined. The different steps may be listed as follows:

1. Determine the most common width barns.

2. Determine stable shapes for the two and three rafter gambrel barn roofs.

3. Consider the rafters as a three hinged arch.

4. Compare stable shape roofs with Wooley's recommended pitches of 6/7 and 7/24.

5. Determine allowable stresses.

6. Analyze the two rafter gambrel barn roofs for dead load and for combined dead and wind loads, with wind from the end and from the side.

7. Analyze the three rafter gambrel barn roofs for dead load and for combined dead and wind loads, with wind from the end and from the side.

8. Draw plans for the designed rafters.

Size of barns to be used

As previously states, the 32', 34', 36' and 40' barns are the ones most commonly found in Iowa. These are the four widths of barns used in this investigation.

<u>Stable shapes for gambrel barn roofs</u>

<u>Desirability of stable roof shapes</u>. A roof that has no tendency to sag at the ridge under dead loads is very desirable. For a roof to be stable, the moment at all joints must be negligible under dead loads. It is possible to achieve this by designing the roof in such a manner that the line of resistance passes through all joints. If the line of resistance passes through the joints, there is zero moment at the point and the joints carry axial loads only. The rafter splice will then have no tendency to move in any direction and the forces will have no rotational effect.

If the rafter splice has no tendency to move inward or outward, the roof cannot sag except through bending of the rafter members. Such shapes may be selected that the rafters will carry all the dead load and the braces will carry only wind and hay loads.

<u>Selection of rafter lengths</u>. For each barn width and each combination of rafter lengths, there is only one shape that is stable under all dead loads. In this investigation standard lengths of material were used for each combination. By using standard lengths it is possible to secure economical designs and to reduce waste to a minimum.

Each stable shape was determined on the center line of the rafter members. The center line distance is approximately two inches shorter than the length of the outside edge for each member. One inch was allowed for sawing of each rafter member. This made the center line length three inches less than the length of the original member. Each barn width was taken as six inches less than the full width.

Method of solution. The determinations were made graphically as shown in Figures 6 and 7. The method used was similar to the methods employed by Pickard (22) and Rice (23). The loads were taken in terms of W, where W is equal to the weight of one square foot of the roof. Any change in the weight of the roof has no effect on the stable shape, provided the change is the same at all points. Rafters were assumed to be spaced on two foot centers, giving a load of 2W per linear foot for each member.

In the determination of the stable shape roofs, the width of the barn, the length of the rafter members, and the vertical forces or dead loads were known. The unknown component was the magnitude of the reaction R_2, which is normal to the load line at the ridge. Trial poles for the force polygon were taken along the line of action of R_2. The line of resistance, being the component of the assumed force R_2 and the vertical force acting at the ridge, was started at that point. The length of the line of resistance between loads was the length of the corresponding rafter section. The application of the vertical load, which acts at the

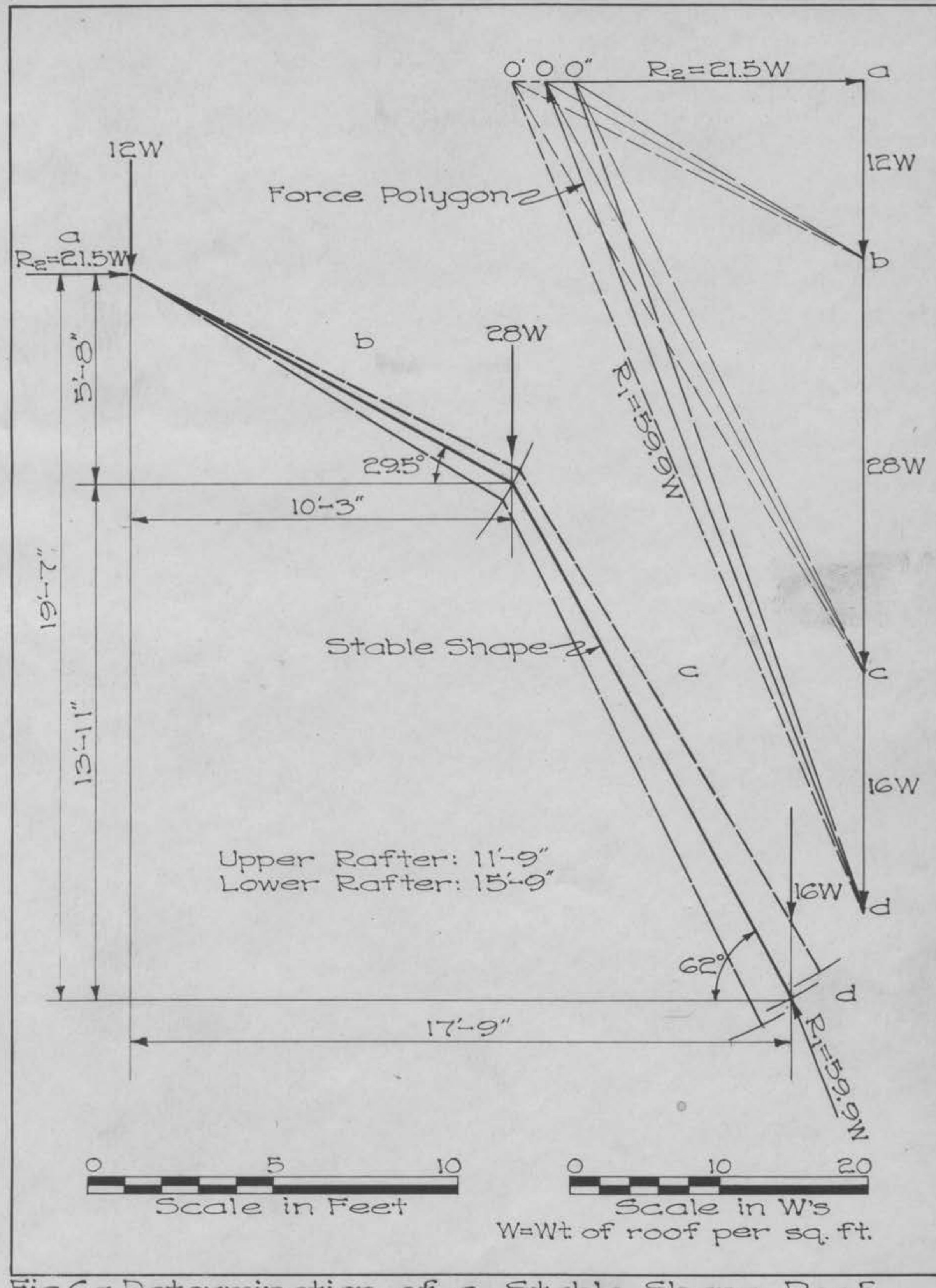


Fig.6- Determination of a Stable Shape Roof.
36' width- 2 rafter- Dead load.

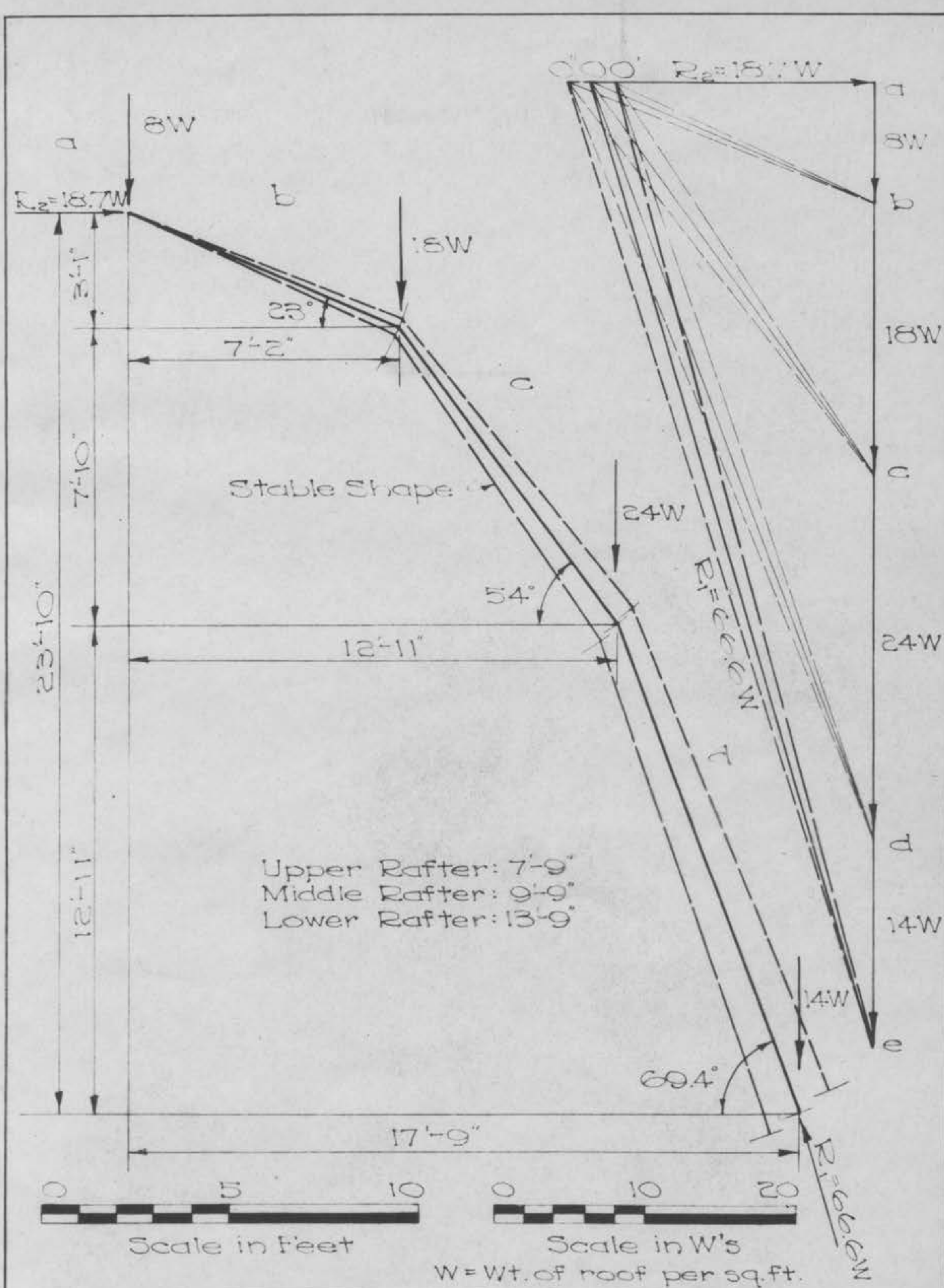


Fig. 7 -Determination of a Stable Shape Roof.
36' width - 3 rafter - Dead Load

joint between the first and second member, changed the direction of the line of resistance of the second member. The length of the line of resistance for the second rafter member was equal to the length of the corresponding rafter section chosen for that position. In the case of the three rafter gambrel barn roofs, there were three rafter members and the line of resistance for the third member was chosen in the same manner. The line of resistance that ends on the center line of the stud is the stable roof shape for that particular combination of rafter sections and barn width.

Stable shapes determined. In this part of the investigation, 21 stable shapes for two rafter gambrel barn roofs were determined. Table 1 gives the width barn, height from plate to ridge, area of cross section, length of rafter members, and the pitch of rafter members for these stable shapes. Table 2 gives the same information for 24 stable shapes for three rafter gambrel barn roofs.

Although some of these shapes are not well proportioned and may not be particularly pleasing in appearance, they may be useful in many instances. Very often a farmer already has material of certain lengths on hand and he does not wish to purchase new lumber. By the use of these tables he can find information for many combinations of rafters for stable roof shapes, and the chances are that they will meet his need. The tables also indicate that it is possible to secure a

Table 1. Stable Roof Shapes for Two Rafter Gambrel Barn Roofs Using Standard Length Material

Width of Barn	Height from Plate to Ridge	Area of Cross Section	Length of Rafters on Center Line		Rise of Rafters Per 12 in. Run	
			Upper	Lower	Upper	Lower
ft.	ft. - in.	sq. ft.	ft.-in.	ft.-in.	in.	in.
32	14 - 7	299.5	9 - 9	11 - 9	5-1/8	17-13/16
32	16 - 6	329.2	11 - 9	11 - 9	7-1/4	21-3/4
32	19 - 0	378.9	11 - 9	13 - 9	8-1/16	25-3/4
32	16 - 3	329.6	9 - 9	13 - 9	6	20-3/4
32	21 - 4	435.0	11 - 9	15 - 9	8-3/8	28-3/16
34	23 - 3	492.5	13 - 9	15 - 9	9-1/2	29-11/16
34	18 - 2	382.6	11 - 9	13 - 9	6-15/16	21-11/16
34	15 - 5	323.4	11 - 9	11 - 9	6-3/8	19-5/8
34	20 - 7	441.0	11 - 9	15 - 9	7-5/8	24-11/16
34	20 - 8	435.6	13 - 9	13 - 9	8-11/16	25-3/4
36	22 - 5	503.5	13 - 9	15 - 9	8-3/4	26-3/8
36	19 - 10	445.4	11 - 9	15 - 9	6-13/16	22-9/16
36	17 - 10	383.8	9 - 9	15 - 9	4-3/4	18-3/8
36	19 - 8	436.1	13 - 9	13 - 9	7-13/16	23-1/16
36	17 - 1	382.9	11 - 9	13 - 9	6-1/8	19-3/16
40	14 - 9	365.9	11 - 9	13 - 9	4-3/8	14-5/16
40	17 - 8	445.5	13 - 9	13 - 9	6-1/8	18-1/2
40	17 - 8	446.6	11 - 9	15 - 9	5-1/8	17-13/16
40	23 - 2	584.4	15 - 9	15 - 9	8-1/8	24-5/8
40	22 - 11	579.0	13 - 9	17 - 9	7-1/4	23-9/16
40	20 - 6	520.6	13 - 9	15 - 9	7-1/16	20-3/4

Table 2. Stable Roof Shapes for Three Rafter Gambrel Barn Roofs Using Standard Length Material

Width of Barn	Height from Plate to Ridge	Area of Cross Section	Length of Rafters on Center Line			Rise of Rafters Per 12 in. Run		
			Upper	Middle	Lower	Upper	Middle	Lower
ft.	ft. - in.	sq. ft.	ft.-in.	ft.-in.	ft.-in.	in.	in.	in.
32	20 - 7	434.7	7 - 9	7 - 9	11 - 9	5-9/16	16-13/16	31-1/4
32	18 - 3	385.0	7 - 9	7 - 9	9 - 9	5-1/4	15-5/8	27-1/16
32	20 - 8	441.2	7 - 9	9 - 9	9 - 9	5-9/16	17-13/16	31-5/8
32	22 - 10	491.0	7 - 9	9 - 9	11 - 9	6	19-3/16	34-7/8
34	19 - 10	449.2	7 - 9	9 - 9	9 - 9	5-1/8	16-3/16	28-15/16
34	22 - 2	504.4	7 - 9	9 - 9	11 - 9	5-3/8	17-7/16	31-1/4
34	24 - 6	553.2	7 - 9	9 - 9	13 - 9	5-9/16	17-1/8	34-7/8
34	24 - 7	555.8	7 - 9	11 - 9	11 - 9	5-3/4	20	36-7/8
34	24 - 8	556.7	9 - 9	9 - 9	11 - 9	7-3/16	21-3/16	36-7/8
36	21 - 5	513.6	7 - 9	9 - 9	11 - 9	4-11/16	15-3/8	28-3/16
36	24 - 0	574.7	9 - 9	9 - 9	11 - 9	6-3/8	19-3/16	33
36	26 - 0	627.1	7 - 9	11 - 9	13 - 9	5-7/16	18-7/16	34-7/8
36	23 - 9	568.8	7 - 9	11 - 9	11 - 9	5-1/8	17-13/16	33
36	23 - 10	577.9	7 - 9	9 - 9	13 - 9	5-1/4	16-1/2	31-1/4
36	26 - 3	631.0	9 - 9	9 - 9	13 - 9	6-13/16	20	35-7/8
36	26 - 3	630.0	9 - 9	11 - 9	11 - 9	6-13/16	20	36-7/8
40	19 - 8	516.6	7 - 9	9 - 9	11 - 9	3-13/16	12-3/16	22-9/16
40	22 - 4	588.9	9 - 9	9 - 9	11 - 9	5-1/4	15-5/8	27-1/16
40	24 - 10	656.6	9 - 9	11 - 9	11 - 9	5-9/16	17-13/16	31-1/4
40	27 - 0	725.7	9 - 9	11 - 9	13 - 9	6	18-7/16	33
40	24 - 8	680.0	9 - 9	9 - 9	13 - 9	5-5/16	16-1/2	28-3/16
40	29 - 6	792.9	9 - 9	13 - 9	13 - 9	6-1/8	20-3/4	38-1/16
40	29 - 4	791.0	9 - 9	11 - 9	15 - 9	6-1/8	19-3/16	34-7/8
40	24 - 8	669.9	7 - 9	11 - 9	13 - 9	4-1/4	15-1/4	29-1/8

large variety of sizes for the cross sectional area of mows, using stable roof shapes.

Rafter considered as a three hinged arch

All rafters analyzed in this investigation were considered as a three hinged arch with the ends hinged at the plate and ridge. This assumption was made in view of the fact that it is practically impossible to obtain a rigid connection at the plate and ridge. Nails, as pointed out previously, deflect immediately upon applying a small load. Timber connectors and bolts give almost a perfect hinged joint. Only through the use of glue is it possible to secure a joint that approaches rigid framing.

All roofs analyzed in this investigation were assumed to be constructed of 2"x6" material. This assumption was based on the fact that the vast majority of the barn roofs are constructed of this size material. In the final design the size of material was changed to conform with the results of the stress analyses.

Selection of loads

Dead load. The dead loads used in this investigation are 1.4 #/sq. ft. for sheathing, 2.5 #/sq. ft. for roofing, and 1.1 #/sq. ft. for 2"x6" rafter members spaced on 2' centers. In calculating the dead loads the wood was assumed to weight

35 lbs. per cu. ft. This is approximately what the average material used in farm building construction weighs. The total dead load is 5 lbs. per sq. ft.

Wind load. The wind load used in this investigation is the load produced by a 70 M.P.H wind. The pressure in lbs. per sq. ft. was calculated by the use of the formula,

$$P = 0.001189 \left(V \frac{22}{15}\right)^2,$$ where V is the velocity of the wind in miles per hour.

The wind pressure distribution diagrams used in this investigation were designed by Pickard (22) and Rice (23). The designs were approved by Dr. Hugh L. Dryden of the United States Bureau of Standards.

Barn Roofs investigated.

Each of the barn roofs analyzed in this study has a stable shape under all dead loads. The roofs are well proportioned and, in general, meet the aesthetic requirements of barn framing. The investigation includes an analysis of 2 - 32' barn roofs, 2 -34' barn roofs, and 2 - 36' barn roofs of the two rafter gambrel barn roof type. Of the three gambrel barn roof type, 2 -36' barn roofs and 2 - 40' barn roofs were analyzed. Information regarding the pitch of these rafters is included in Tables 1 and 2.

Method of analysis

The method used in analyzing the barn roofs is that of graphic statics as outlined by Fuller and Kerekes (13) in their text, "Analysis and Design of Steel Structures."

The three conditions of equilibrium of a three hinged arch $\sum F_h = 0, \sum F_v = 0$, and $\sum M = 0$ may be expressed graphically. Since a force is completely defined when its magnitude, line-of-action, and direction are known, it is possible to represent any force by a line. The length of the line represents the magnitude of the force to some suitable scale; the position of the line represents the line-of-action of the force; and the arrow head at the end of the line represents the direction in which the force is acting.

Every force, represented graphically, may be resolved into its vertical and horizontal components. Instead of working with these components, the forces may be drawn directly so that one force begins where the other force ends. The resulting diagram is called a force polygon when the final force ends where the first force begins. The closure of the force polygon indicates that $\sum F_h = 0$ and $\sum F_v = 0$.

A graphic determination of the third condition of equilibrium $\sum M = 0$ may be made by construction of an equilibrium polygon. A fofce may be replaced by any two components whose lines-of-action intersect anywhere on the line of action of the original force. Likewise, a force may be held in equilibrium by two other forces whose lines-of-action intersect on the line of action of the original force.

To construct an equilibrium polygon, a force polygon is first constructed for all the external forces acting on the structure. Each of these forces may be held in equilibrium by a pair of equilibrants, so chosen that the equilibrants of all the external forces intersect at a common point called the Pole. The equilibrants are then transferred to the space diagram; each pair of equilibrants intersects on its own force and each equilibrant, being common to two adjacent forces, occupies the same line-of-action in the space diagram. If the equilibrants form a closed polygon, the external forces are in equilibrium, because each side of the equilibrium polygon represents two equal and opposite forces acting along the same line-of-action.

In order to study how the applied loads are transmitted to the supports within the limits of a definite structure, it becomes necessary to pass an equilibrium polygon through a point at each support and a third point that lies somewhere between the supports. In this investigation of gambrel barn roofs, the two points of support are the plates and the third point is the ridge. After the construction of the equilibrium polygon through the three points it is then possible to determine the moment about any point on the frame. This is done by measuring the distance from a given point on the frame perpendicular to the equilibrant corresponding to that point, and then multiplying this distance by the thrust of the equilibrant encountered. The product of the thrust times the distance is the moment about the point considered.

By the use of the equilibrium polygon it is also possible to determine the vertical shear and the direct stress in the members at various points. This is accomplished by determining the component of the equilibrant perpendicular to the member considered for vertical shear and the component parallel to the member for direct stress.

The unit fiber stress due to bending moment was calculated by the formula $S = \frac{MC}{I}$, where S is the unit stress, M the moment, C the distance from the neutral axis to the outmost fiber, and I the moment of inertia of the cross section about its neutral axis. The unit horizontal shear was calculated by the formula $v = \frac{VQ}{It}$, where v is the unit horizontal shear, V the total shear, Q the moment of the area between the extreme fiber and the point at which the horizontal shear is desired, I the moment of inertia of the cross section about its neutral axis, and t the thickness of the beam. The unit stress due to direct loading was found by dividing the total stress by the area of the cross section of the member.

Allowable working stresses

The allowable fiber stress selected for use in design of the rafter members was 3000 #/sq.in. This stress was used only in designing for combined dead and wind loads. The modulus of rupture for southern yellow pine is 12,800 #/sq. in. (27) and for Douglas fir this value is 11,700 #/sq.in. If 3000 #/sq.in. is used, there is still a factor of safety of 4.26 for yellow pine and 3.9 for Douglas fir; therefore, it seems that 3000 #/sq.in. is a reasonable value. Also, in case one rafter member is unsound, the load can be transferred to the other members without fear of failure.

The allowable shear value was taken as 120# /sq.in. The allowable compressive stress parallel to the fiber was taken as 1,200 # / sq.in.

Stable roof shapes and Wooley's recommended pitches of 6/7 and 7/24

Until the last few years, stable roof shapes were unknown. Various pitches were used and recommended for use in the construction of barns. Wooley (29) points out that the pitches 6/7 and 7/24 are to be recommended for use in construction of gambrel barn roofs, as these pitches come relatively close to a line of an inverted catenary. However, by definition, a catenary is the position assumed by a perfectly flexible cord or chain hanging freely between two points of support. The requirement of forces, in order that the line of resistance will form an inverted catenary, is that they must be distributed uniformly along a line, which is not the case of a gambrel barn roof. If the rafter members of the gambrel barn roof were infinitely short in length, then the line of resistance would approach a catenary.

In an effort to determine the difference in Wooley's recommended pitches of 6/7 and 7/24 and stable roof shapes, drawings were made of gambrel barn roofs, using these pitches. Then, through the same points at the plate and ridge, stable roof shapes were drawn. These shapes showed the actual line of resistance that the forces should follow. This procedure was followed on the 32',34',36'&40' widths of the two rafter gambrel roof barns. Figures 8, 9, and 10 show a graphical comparison of the pitches and stable shapes. In each case the line of resistance of the stable shape fell outside the line the pitches followed. In case a relatively heavy hay track is fastened at the ridge, it might be an advantage to have a roof shape with the line of resistance slightly outside. The hay track would give a slightly larger vertical load which would tend to make the slppe of the line of resistance a little greater.

Moments at the rafter joints were calculated algebraically for gambrel barn roofs using the recommended pitches. Table 3 shows the results of this investigation. The greatest moment was 1808 in. lbs. on the 40' barn. This moment, although not very large, is capable of causing deflection at

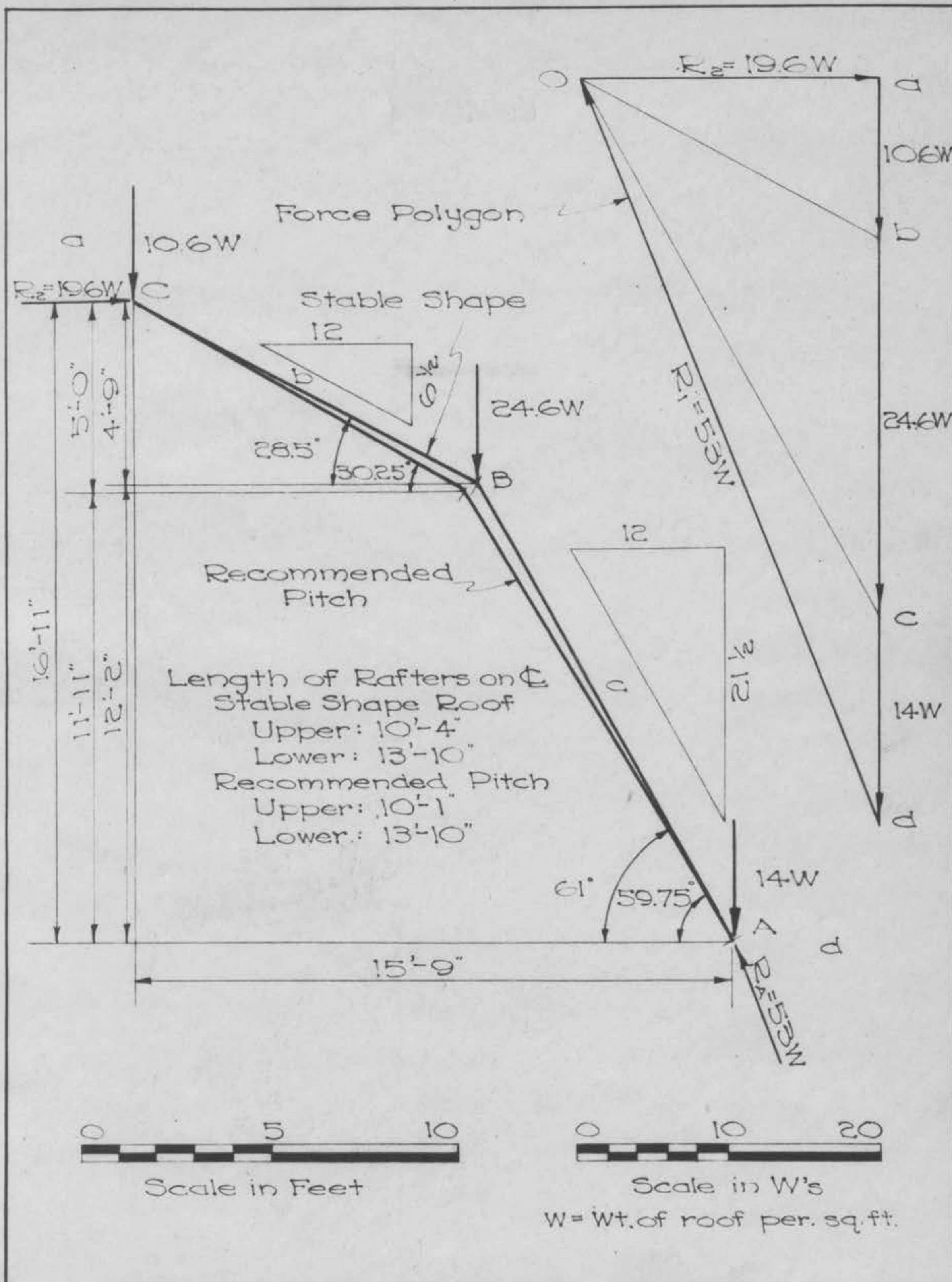


Fig. 8 - Comparison of Stable Shape with Wooley's Recommended Pitches of 6/7 and 7/24.
32' width - 2 rafter - Dead load.

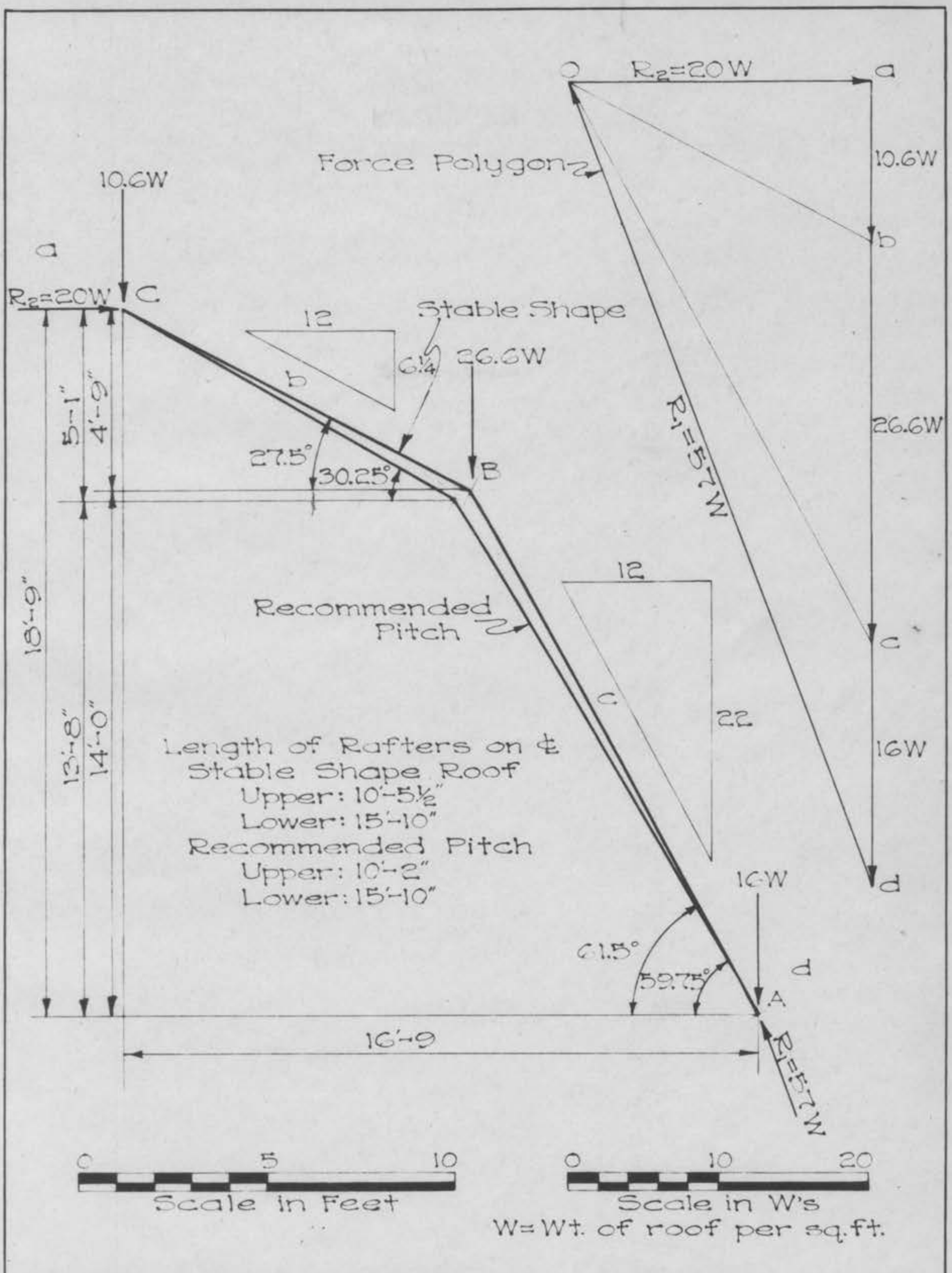


Fig. 9-Comparison of Stable Shape with Wooley's Recommended Pitches of 6/4 and 7/24.
34' width - 2 rafter - Dead load

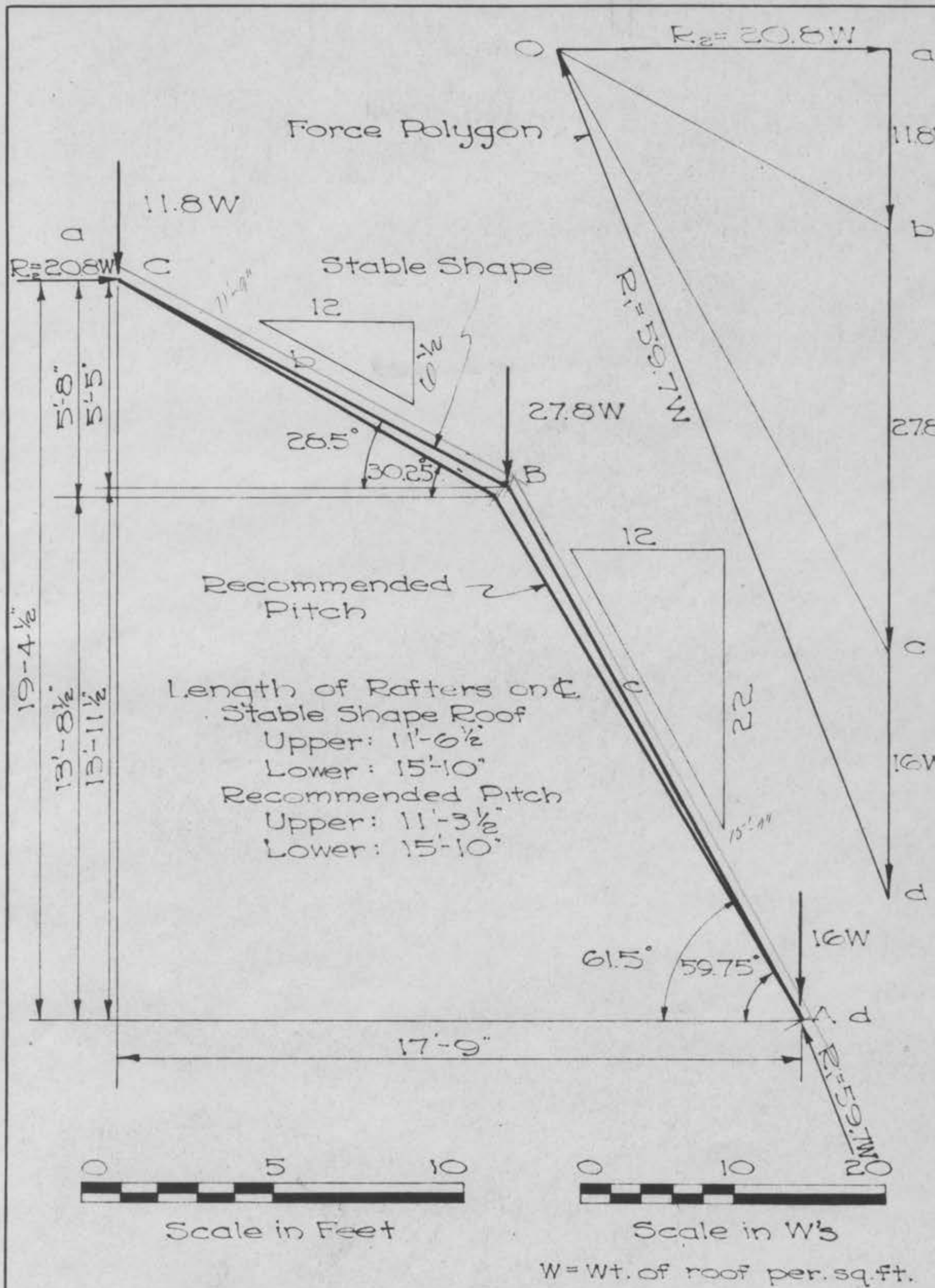


Fig. 10 Comparison of Stable Shape with Wooley's Recommended Pitches of 6/7 and 7/24.
36' width - 2 rafter - Dead load.

the joint. This is especially true if the members are connected by nails (22).

Table 3. Moments at Rafter Joints
Using 6/7 and 7/24 Pitches

Width of Barn feet	Moment in.lbs.
32	524
34	780
36	725
40	1808

Another objection to the above recommended pitches is that they do not utilize standard length lumber. This necessitates the loss of some material due to sawing, which increases the cost of the building.

After this investigation it was believed that a complete stress analysis of a stable roof shape and a roof constructed with the above pitches should be made. Bending moments were calculated algebraically at each of 11 points along the rafter members for both roofs. A summary of the bending moments, shears, and stresses is shown in Table 4. Figure 11 represents a graphic analysis of the results.

Results. The maximum bending moment in the lower rafter member for Wooley's pitches was 2,400 in.lbs. The maximum bending moment in the lower rafter member for the stable roof

Table 4. Summary of Moments, Shears, and Stresses
Rafter Under Dead Load Hinged at Plate and Ridge
Wooley's Recommended Pitches of 6/7 and 7/24
34' Barn with 2"x6" x 10'-4" & 16' Rafter Members

Pt.	e in.	Thrust #	Moment in. #	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Direct Stress #	Direct Fiber Stress #/sq."
A	0.00	285*	0	0.0	45	7.39	281	30.75
1	3.68	285	+1,050	122.7	45	7.39	281	30.75
2	8.88	248	+2,200	257.0	24	3.94	247	27.05
3	11.42	212	+2,400	280.2	4	0.66	212	23.20
4	9.00	178	+1,600	186.9	16	2.63	177	19.40
B	5.27	148	+ 780	91.1	36	5.91	144	15.77
5	10.88	148	+1,610	188.0	39	6.41	142	15.55
6	15.23	130	+1,930	231.2	15	2.46	129	14.12
7	14.58	116	+1,692	197.7	6	0.98	116	12.70
8	7.65	104 107	+ 795	92.8	30	4.93	103	11.28
C	0.00	104	0	0.0	52	8.54	90	9.85

Horizontal Reaction A = 104# Vertical Reaction A = 264#

Table 5. Summary of Moments, Shears, and Stresses
Rafter Under Dead Load Hinged at Plate and Ridge
Stable Roof Shape for Same Barn as Above
with Ridge of Equal Height
34' Barn with 2"x6" x 10' - 7-1/2" x 16'

A	0.00	286	0	0.0	40	6.57	283	31.00
1	3.08	286	+ 883	103.1	40	6.57	283	31.00
2	7.74	248	+1,920	224.1	20	3.29	248	27.16
3	8.98	214	+1,920	224.1	1	0.16	213	23.32
4	6.01	180	+1,080	126.1	18	2.96	179	19.60
B	0.00	148	0	0.0	38	6.24	143	15.66
5	4.83	148	715	83.5	48	7.88	141	15.43
6	11.28	130	+1,465	171.1	24	3.84	128	14.02
7	12.72	116	+1,473	172.2	1	0.16	116	12.70
8	7.21	102	+ 736	85.9	22	3.61	104	11.39
C	0.00	102	0	0.0	46	7.56	91	9.96

Horizontal Reaction A = 102# Vertical Reaction A = 268#

*Positive sign denotes tension on inner fiber.

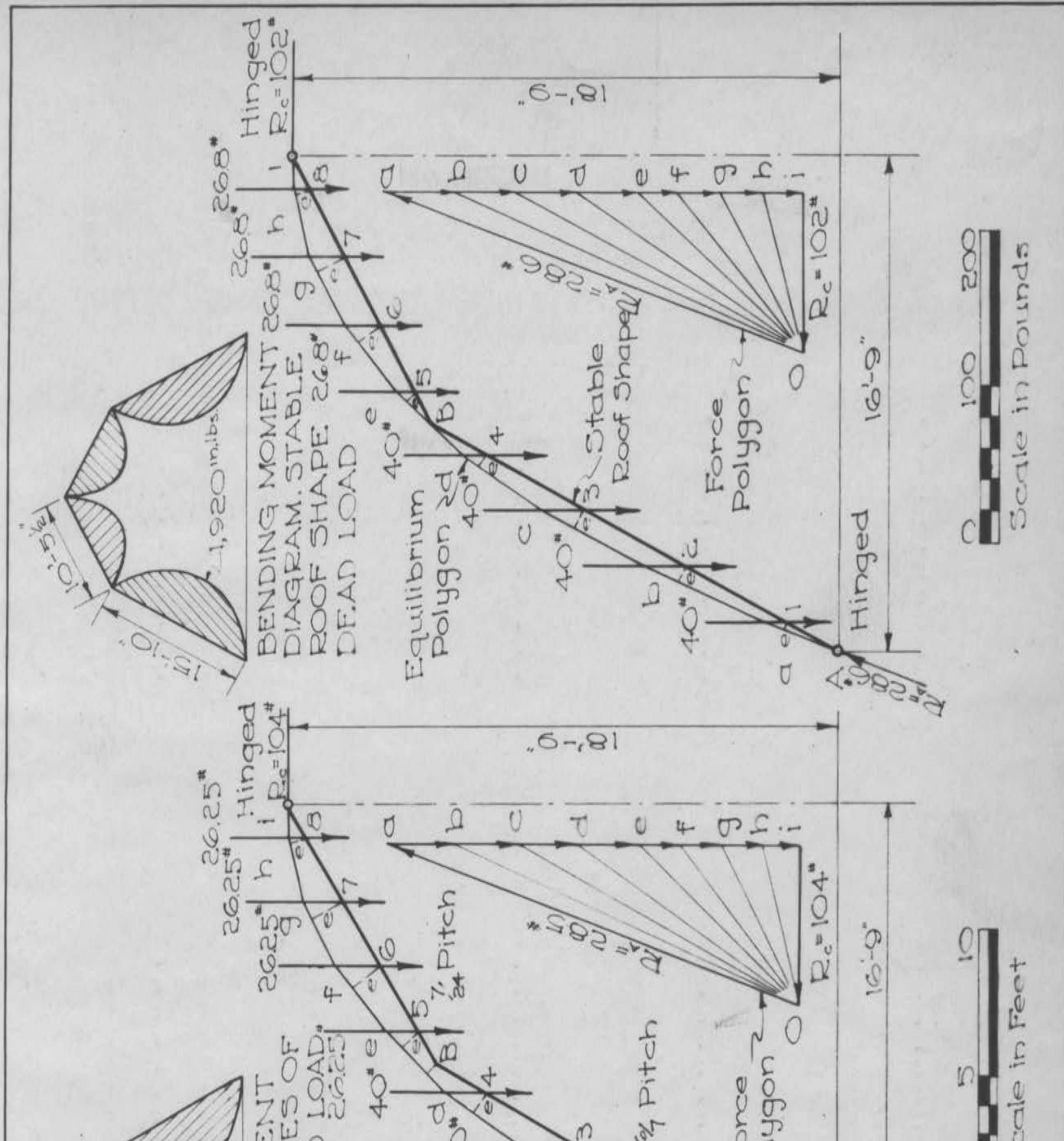


Fig. 11 - Stress Analysis - Wooley's Pitches - Stable Shape
34' width - 2 rafter - Dead load.

shape was 1,920 in.lbs. The maximum bending moment in the upper rafter member for Wooley's pitches was 1,930 in.lbs. The maximum bending moment in the upper rafter member for the stable roof shape was 1,473 in. lbs. The horizontal shear and the direct stress were approximately the same for Wooley's pitches and the stable roof shape.

These results indicate clearly the reason for sagging along the ridge. Not only is there moment at the joints of the rafters, when the recommended pitches are used, but there is a considerably larger moment at the middle of the rafter member, as compared to the moment at the same point on the stable roof shape.

Conclusions.

1. Standard pitches do not utilize standard lengths of material in most cases.

2. Stable roof shapes utilize standard lengths of material.

3. For the same height and width of barn the stable roof shape furnishes more mow space.

4. Bending moment at the joints of the rafter members, resulting from the use of Wooley's pitches of 6/7 and 7/24, may cause deflection of the joints which result in sagging along the ridge.

5. The bending moment is larger in the middle of the rafter members of the recommended pitches than it is at the

corresponding point on the stable roof shape.

Stress analyses of two rafter gambrel barn roofs

The method used in making analyses of the barn roofs in this investigation is described on page 45. The procedure may be summarized as follows:

1. Roof loads were determined.

2. A space diagram of the roof was constructed.

3. Loads were placed at the center of each of four equal divisions of each rafter member and a force polygon was constructed using the determined loads.

4. An equilibrium polygon was passed through the three hinged points.

5. Reactions at the hinged points were determined.

6. Eccentricities and thrusts were scaled and the moments calculated at the points of loading and at each rafter joint.

7. Vertical shear and direct stresses were scaled.

8. Unit stresses due to moment, shear, and direct stress were calculated.

Dead load stress analyses of the 34' and 36' barn roofs

34' barn with 14' and 12' rafters. A graphic stress analysis of this barn is presented in Figure 12. Table 6 shows a summary of the moments, shears, and stresses taken

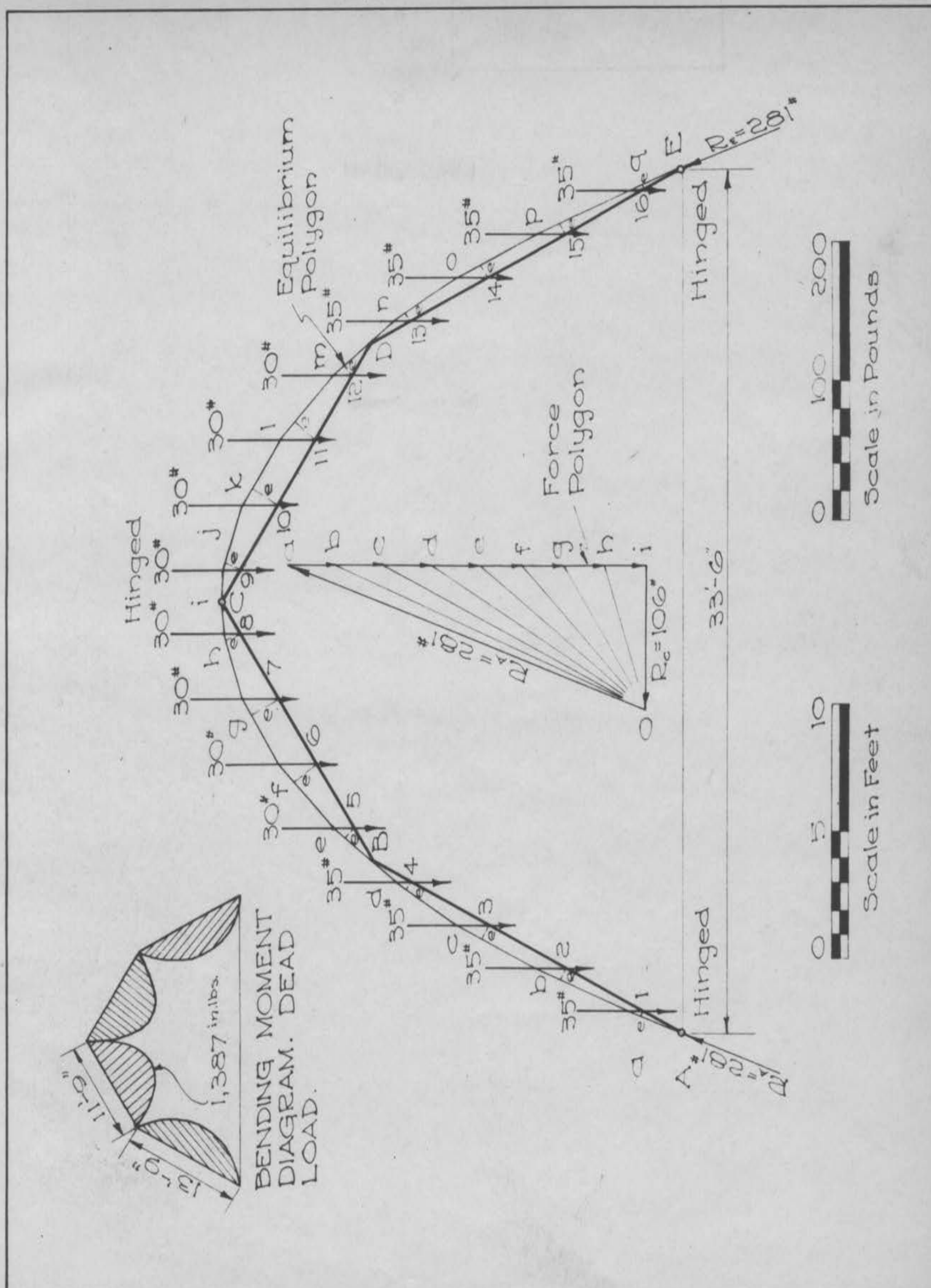


Fig.12 - Stress Analysis - Dead Load
34' width - 2 rafter - Stable shape

Table 6. Summary of Moments, Shears, and Stresses
Rafter Under Dead Load
34' Barn with 2" x6" x 14' & 12' Rafter Members
Hinged at Plate and Ridge

Pt.	e in.	Thrust #	Moment in.#	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Direct Stress #	Direct Fiber Stress #/sq."
A	0	281	0	0	34	5.59	279	30.54
1	2.78	281	+ 753	87.9	34	5.59	279	30.54
2	5.57	249	+1,387	162.0	16	2.63	248	27.15
3	6.18	218	+1,348	157.4	0	0	218	23.84
4	4.02	189	+ 760	88.7	17	2.79	187	20.26
B	0	161	0	0	52	8.54	153	16.74
5	6.62	161	+1,066	123.3	52	8.54	153	16.74
6	13.67	140	+1,915	223.6	27	4.43	139	15.21
7	15.68	122	+1,910	223.0	0	0	122	13.35
8	8.20	110	+ 869	101.6	25	4.11	107	11.72
C	0	106	0	0	52	8.54	92	10.07

Horizontal Reaction A = 106# Vertical Reaction A = 260#

Table 7. Summary of Moments, Shears, and Stresses
Rafter Under Dead Load
36' Barn with 2"x6" x 16' & 12' Rafter Members
Hinged at Plate and Ridge

A	0	301	0	0	36	5.81	298	32.60
1	2.86	301	+ 860	100.5	36	5.81	298	32.60
2	6.77	264	+1,788	208.7	18	2.96	263	28.80
3	7.82	228	+1,771	207.0	0	0	228	24.96
4	4.34	194	+ 841	97.3	20	3.29	192	21.10
B	0	163	0		52	8.54	154	16.86
5	5.86	163	955	111.5	52	8.54	154	16.86
6	13.50	141	+1,905	222.5	24	3.94	140	15.32
7	15.32	124	+1,900	221.8	0	0	124	13.58
8	8.34	113	+ 940	109.8	26	4.27	109	11.92
C	0	108	0	0	52	8.54	94	10.28

Horizontal Reaction A = 108# Vertical Reaction A = 280#

from this analysis. The moments in this analysis were all calculated algebraically and checked graphically.

The maximum bending moment found was 1,915 in.lbs at point 6. This moment produces a fiber stress of 223.6 #/sq.in., which is well under the allowable working value. No moment is created at the rafter joint as a stable roof shape was used in the analysis.

The maximum horizontal shear is 8.54 #/sq.in. Since the allowable value varies from 120 #/sq.in. to 150 #/sq.in., this is of no concern in a roof subjected to dead load.

The maximum direct fiber stress is 30.54 #/sq.in. at the plate. The allowable value is 1,200 #/sq.in.; hence, this has no effect on the design.

The horizontal reaction at the plate (point A) is 106# and the vertical reaction at the same point is 260#, which is equal to the dead load on that side.

<u>36' barn with 16' and 12' rafters</u>. A graphic stress analysis of this barn is presented in Figure 13. Table 7 shows a summary of the moments, shears, and stresses determined in this analysis. The moments in this analysis were calculated algebraically and checked graphically.

The maximum bending moment found in the analysis was 1,905 in. lbs. at point 6. This value is slightly less than the maximum bending moment found in the analysis of the 34' barn. This decrease was due to the increase in the inclination

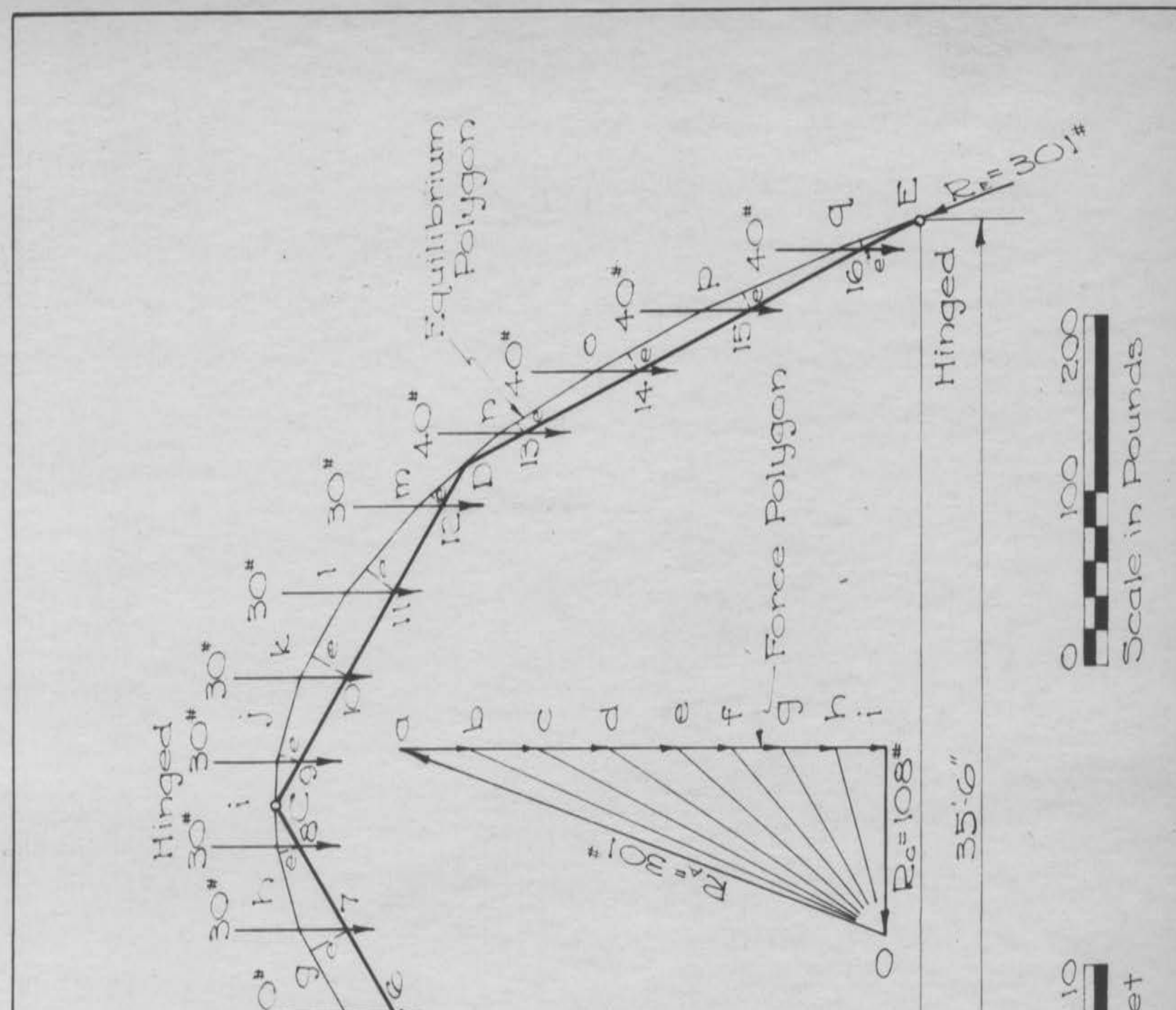


Fig.13 - Stress Analysis - Dead Load
36' width - 2 rafter - Stable shape

of angle of the rafter with the horinzontal, thereby decreasing the horizontal component of the rafter length. The maximum bending moment in the lower rafter member was found to be 1,788 in. lbs. This is a 29 per cent increase in the moment found at the corresponding point on the 34' barn roof. Again, no moment was found at the rafter joint. The maximum fiber stress created by the bending moment is 222.5 #/sq.in.

The maximum horizontal shear is 8.54 #/sq.in. This again indicates that the horizontal shear is of no concern in designing a roof subjected to dead load.

The maximum direct fiber stress is 32.6 #/sq.in., which is of no concern in this analysis.

The horizontal reaction at the plate (point A) is 108# and the vertical reaction is 280#.

Conclusions.

1. Moments, shears, and stresses created by dead loads on stable shape roofs are not important factors in design.

2. The bending moment at all joints of stable roof shapes is zero under dead load.

3. Reactions at the plates of roofs subjected to dead load are of little concern in designing the connection between the rafter and the plate and studding.

Combined dead and wind load stress analysis, 32' barn with 14' and 10' rafter members

The wind load data used in the stress analyses of the 32' barn roofs are shown in Table 8. The wind load and the dead load are combined into resultants acting at four equally divided spaces on each rafter member.

Side wind. A graphical stress analysis of this barn roof is shown in Figure 14. Table 9 shows a summary of the moments, shears, and stresses determined in this analysis.

The maximum bending moment found was 14,760 in. lbs. at point 3 on the windward side. This moment produces a fiber stress of 1,724 #/sq.in., which is considerably under the allowable of 3,000 #/sq.in. However, the 2"x6" member is smallest standard for this barn with the rafters spaced on 2' centers. The use of a 2"x4" member would result in a fiber stress of 4,140 #/sq.in. With the centerline spacing increased to 3', the maximum fiber stress, using the larger member, would be 2,585 #/sq.in., which is well within the allowable working range. The maximum bending moment for the joints is 11,910 in.lbs.

The maximum horizontal shear is 42.7 #/sq.in. Hence, the horizontal shear is not a factor of primary importance in the design of this roof.

The maximum direct stress is 20.1 #/sq.in., which is of

Table 8. Wind Load Data for Two Rafter Gambrel Barn Roofs
32' Barn - 70 M.P.H. Wind

Wind 90° to Side

Load No.	Coefficient	Pressure #/sq.'	14' & 10' Rafter Members Total P #	Dead Load #	Resultant #	14' & 12' Rafter Members Total P #	Dead Load #	Resultant #
1	1.0	12.55	88.0	35	109.6	88.0	35	104.2
2	.965	12.55	85.0	35	103.3	85.0	35	101.5
3	.815	12.55	71.7	35	94.0	71.7	35	88.5
4	.476	12.55	41.9	35	66.5	41.9	35	61.0
5	-.50*	12.55	31.4	25	13.6	37.6	30	21.1
6	-.60	12.55	37.7	25	18.3	45.2	30	26.4
7	-.60	12.55	37.7	25	18.3	45.2	30	26.4
8	-.60	12.55	37.7	25	18.3	45.2	30	26.4
9	-.60	12.55	37.7	25	18.3	45.2	30	26.4
10	-.60	12.55	37.7	25	18.3	45.2	30	26.4
11	-.60	12.55	37.7	25	18.3	45.2	30	26.4
12	-.60	12.55	37.7	25	18.3	45.2	30	26.4
13	-.60	12.55	52.7	35	46.4	52.7	35	49.3
14	-.60	12.55	52.7	35	46.4	52.7	35	49.3
15	-.60	12.55	52.7	35	46.4	52.7	35	49.3
16	-.60	12.55	52.7	35	46.4	52.7	35	49.3

Wind 90° to End

1	-1.6	12.55	141.0	35	127.0	141.0	35	129.8
2	-1.6	12.55	141.0	35	127.0	141.0	35	129.8
3	-1.6	12.55	141.0	35	127.0	141.0	35	129.8
4	-1.6	12.55	141.0	35	127.0	141.0	35	129.8
5	-1.6	12.55	100.3	25	74.0	120.0	30	96.4
6	-1.6	12.55	100.3	25	74.0	120.0	30	96.4
7	-1.6	12.55	100.3	25	74.0	120.0	30	96.4
8	-1.6	12.55	100.3	25	74.0	120.0	30	96.4

*Negative sign denotes suction.

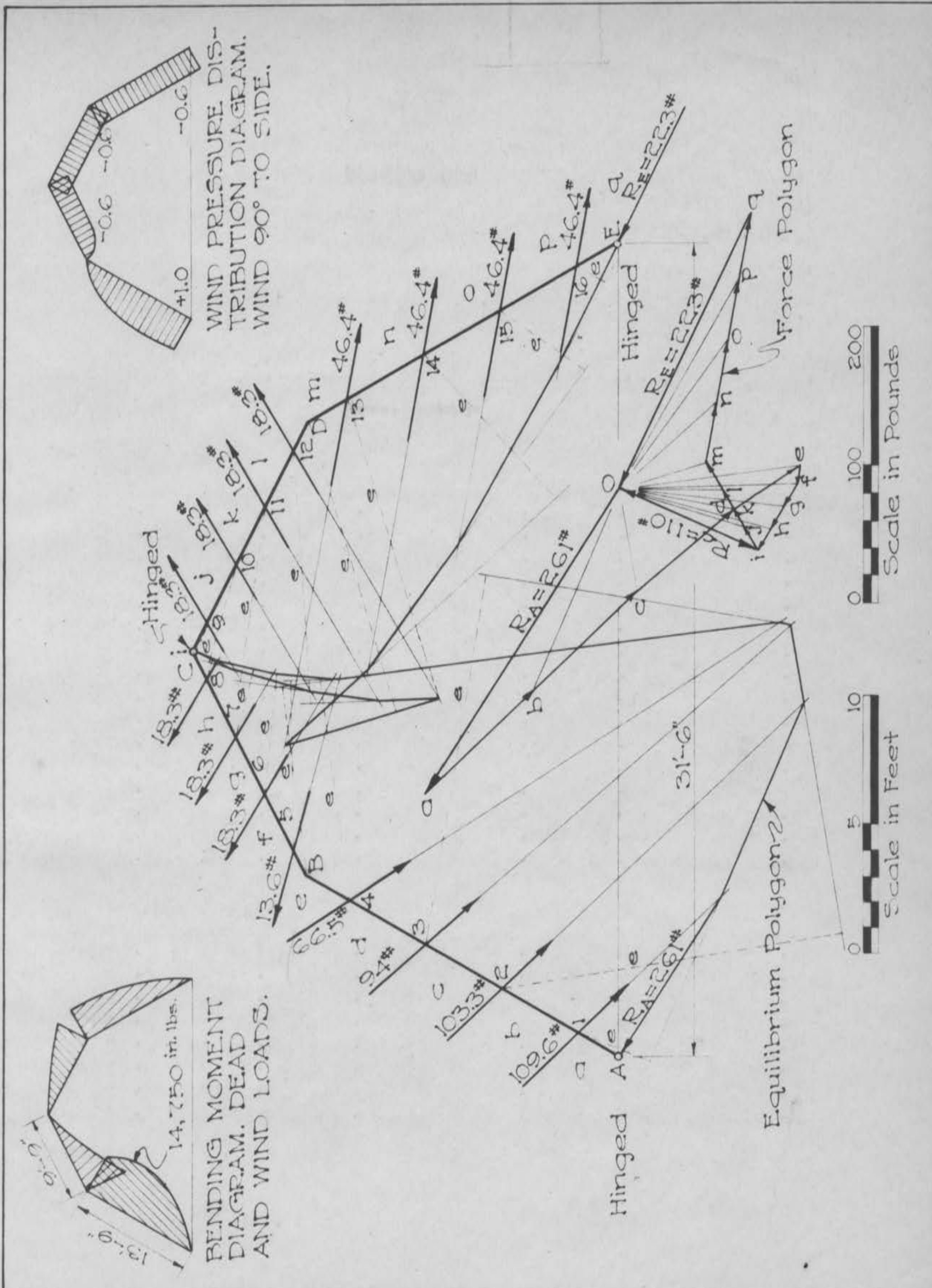


Fig.14 - Stress Analysis-Combined Dead and Wind Loads.
32' width-2 rafter-Side wind.

Table 9. Summary of Moments, Shears, and Stresses
Rafter under Combined Dead and Wind Loads
32' Barn with 2" x 6" x 14' & 10' Rafter Members
Hinged at Plate and Ridge
Wind 90° to side

Pt.	e in.	Thrust #	Moment in. #	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Direct Stress #	Direct Fiber Stress #/sq."
A	0	261	0	0	260	42.70	12	1.31
1	33.0	157	+ 5,180	605	156	25.65	19	2.08
2	160.0	75	+14,750	1,722	59	9.69	49	5.36
3	173.6	85	+14,760	1,724	31	5.09	80	8.76
4	102.4	133	+13,630	1,592	80	13.13	86	9.41
B	89.5	133	+11,910	1,391	125	20.50	44	4.82
5	78.5	129	+10,130	1,183	116	19.05	54	5.91
6	55.2	120	+ 6,625	774	100	16.42	64	7.01
7	32.5	113	+ 3,672	428	85	13.96	77	8.43
8	9.2	110	+ 1,013	118	69	11.32	84	9.20
C	0	110	0	0	109	17.91	5	0.55
9	14.7	110	-1 ,617	189	109	17.91	5	0.55
10	46.8	95	- 4,450	520	93	15.27	15	1.64
11	81.0	83	- 6,725	785	80	13.13	25	2.74
12	118.5	73	- 8,660	1,011	63	10.35	35	3.83
D	140.0	66	- 9,190	1,072	47	7.72	45	4.92
13	145.5	66	- 9,610	1,122	14	2.30	64	7.01
14	90.5	97	- 8,775	1,024	20	3,28	94	10.28
15	48.0	136	- 6,525	762	55	9.03	124	13.58
16	15.4	179	- 2,756	322	90	14.78	154	16.85
E	0	223	0	0	125	20.52	184	20.14

Horizontal Reaction A = 221# Horizontal Reaction E = 204#
Vertical Reaction A = 140# Vertical Reaction E = 97#

Wind 90° to End

Pt.	e in.	Thrust #	Moment in. #	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Direct Stress #	Direct Fiber Stress #/sq."
A	0	407	0	0	310	50.90	266	29.12
1	18.5	349	- 6,460	755	187	30.70	394	32.20
2	42.6	332	-14,140	1,652	64	10.51	326	35.70
3	46.0	361	-16,620	1,942	59	9.69	361	39.52
4	32.8	426	-13,980	1,632	181	29.73	387	42.35
B	24.0	426	-10,220	1,193	57	9.36	422	46.15
5	25.3	432	-10,930	1,277	15	2.46	433	47.40
6	23.3	451	-10,060	1,173	90	14.78	444	48.60
7	16.3	481	- 7,840	915	163	26.78	454	49.70
8	6.0	520	- 3,120	364	235	38.60	463	50.65
C	0	520	0	0	235	38.60	463	50.65

Horizontal Reaction A = 135#
Vertical Reaction A = 386#

no concern in this design.

The horizontal reaction at A is 221# and the vertical reaction at the same point is 140#, which are the maximum reactions for the roof.

End wind. A graphical analysis of the above barn with the wind to the end is shown in Figure 15. Table 9 shows a summary of the moments, shears, and stresses, determined in this analysis.

The maximum bending moment is -16,620 in.lbs. at point 3, which is the same point of the maximum bending moment with the wind to the side. However, with the wind to the end, compression is created on the inner fibers and tension on the outside fibers. The maximum fiber stress created is 1,942 #/sq.in. With the rafters spaced on 3' centers, the fiber stress would be 2,915 #/sq.in., which is under the allowable stress. The maximum bending moment for the rafter joints in this analysis is -10,220 in. lbs.

The maximum horizontal shear is 50.9 #/sq.in. at point A. The shear is again too small to be of importance in this design.

The maximum direct fiber stress is 50.65 #/sq.in., which is also of no concern in the design of this roof.

The horizontal reaction at A is 135#, and the vertical reaction is -386#. In this case the roof has a tendency to lift off of the plate. To counteract this lifting force the

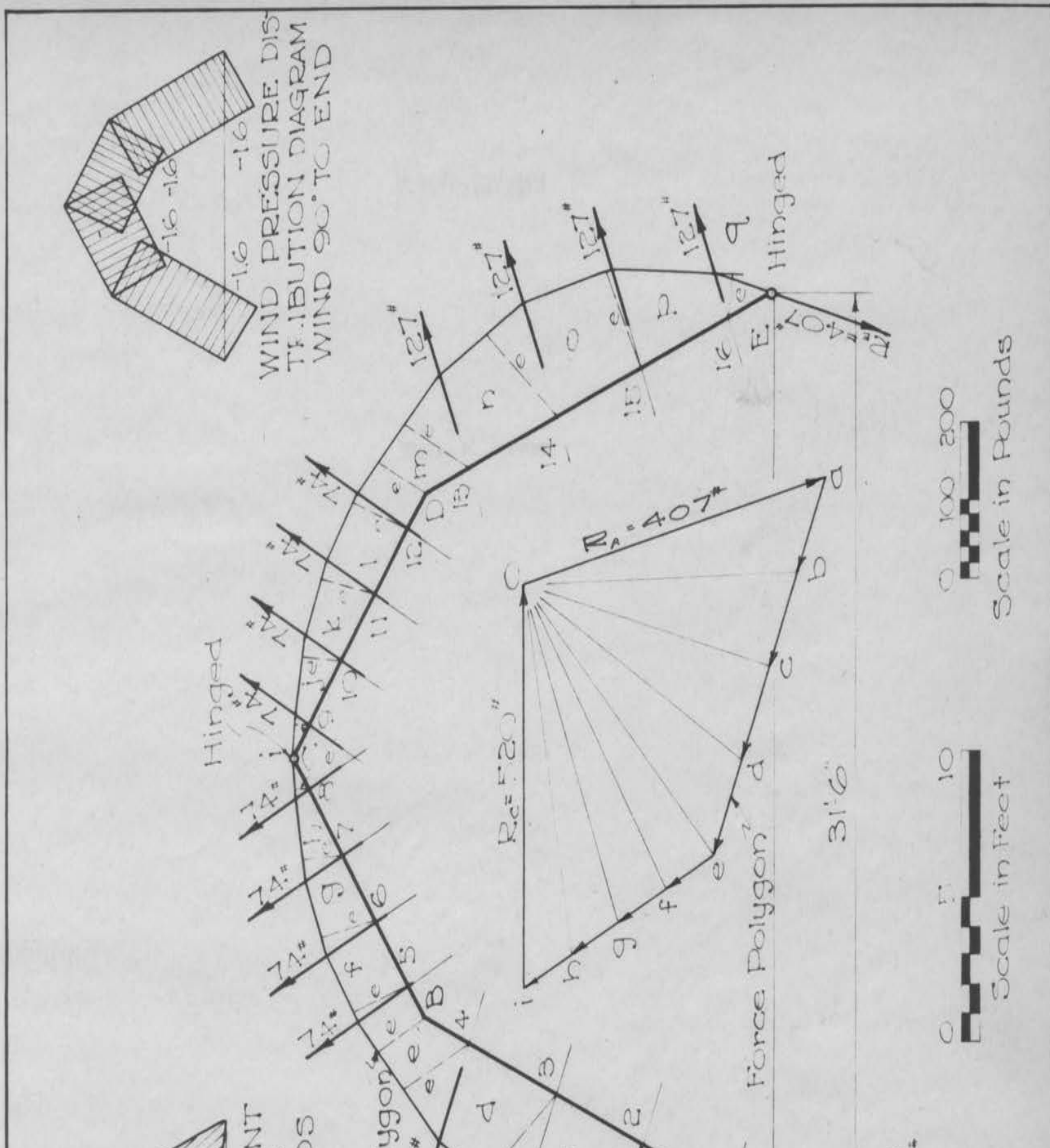


Fig. 15- Stress Analysis -Combined Dead and Wind Loads
32' width - 2 rafter - End wind.

rafter must be securely fastened to the plate and studs. A toe nailed joint consisting of four 16d nails will safely resist a vertical pull of 386#. The holding power of the nails was calculated by using the equation $P = 1150G^{\frac{5}{2}}D$ as shown in "Wood Handbook" (27). The reaction at the ridge (point C) is 520#. This reaction must be considered in designing the rafter tie or collar beam.

Combined dead and wind load stress analysis, 32' barn with 14' and 12' rafter members

Side wind. A graphical stress analysis of this barn roof is shown in Figure 16. Table 10 shows a summary of the moments, shears, and stresses determined in this analysis.

The maximum bending moment found is 14,830 in.lbs. at point 3 on the windward side. This moment produces a fiber stress of 1,733 #/sq.in., which is well under the allowable working stress. However, the 2"x6" member is the smallest standard piece of material that could be used for this barn with the rafters spaced on 2' centers. The use of a 2"x4" member would result in a fiber stress of 4,160 #/sq.in. The maximum bending moment for the joints is 12,350 in. lbs. on the windward side. The bracing or splice used to join the rafter members should start at a point near the center of the lower rafter member (point 2) and run approximately to a

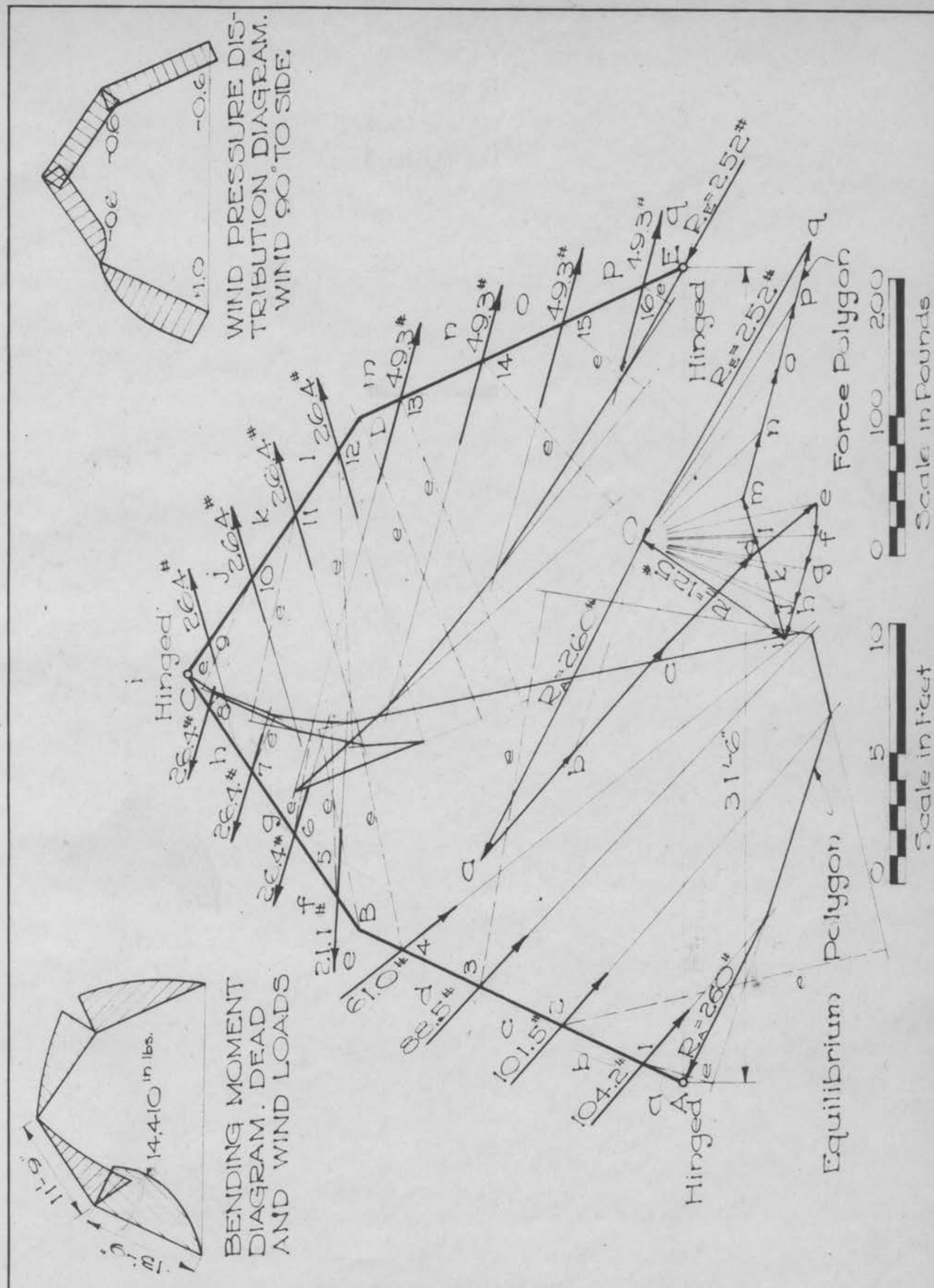


Fig.16-Stress Analysis -Combined Dead and Wind Loads
32' width - 2 rafter - Side wind

Table 10. Summary of Moments, Shears, and Stresses
Rafter Under Combined Dead and Wind Loads
32' Barn with 2"x6" x 14' and 12' Rafter Members
Hinged at Plate and Ridge

Wind 90° to Side

Pt.	e in.	Thrust #	Moment in. #	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Direct Stress #	Direct Fiber Stress #/sq."
A	0	260	0		259	42.55	6	0.66
1	32.8	160	+ 5,250	614	159	26.12	21	2.30
2	153.5	79	+12,120	1,415	62	10.18	49	5.36
3	185.3	80	+14,830	1,733	23	3.78	78	8.54
4	108.0	130	+14,040	1,640	77	12.64	104	11.38
B	95.0	130	+12,350	1,443	119	19.55	50	5.47
5	81.0	126	+10,210	1,192	107	17.58	67	7.33
6	53.5	120	+ 6,360	743	87	14.28	83	9.08
7	27.0	120	+ 3,240	378	67	11.00	100	10.94
8	6.5	125	+ 813	95	47	7.72	117	12.80
C	0	125	0	0	47	7.72	117	12.80
9	17.5	125	- 2,188	256	124	20.37	3	0.33
10	56.2	106	- 5,950	695	105	17.25	16	1.75
11	98.4	91	- 8,950		84	13.80	33	3.61
12	137.2	81	-11,110	1,298	65	10.67	50	5.47
D	150.5	79	-11,900	1,390	44	7.23	66	7.22
13	151.5	79	-11,980	1,400	3	0.49	78	8.53
14	89.8	115	-10,320	1,205	34	5.59	111	12.15
15	47.8	159	- 7,610	889	73	11.99	143	15.65
16	14.7	206	- 3,015	352	112	18.40	175	19.15
E	0	252	0	0	147	24.15	206	22.54

Horizontal Reaction A = 233#   Horizontal Reaction E = 221#
Vertical Reaction A = 116#   Vertical Reaction E = 124#

Wind 90° to End

A	0	435	0	0	356	58.45	247	26.10
1	20.5	363	- 7,440	869	231	37.84	277	30.31
2	51.3	328	-16,830	1,962	106	17.41	309	33.80
3	62.2	343	-21,350	2,495	19	3.12	341	37.30
4	51.0	402	-20,500	2,395	145	23.82	374	40.85
B	44.0	402	-17,610	2,057	67	11.00	396	43.30
5	45.3	415	-18,800	2,195	26	4.27	413	45.20
6	39.7	447	-17,740	2,072	122	20.04	430	47.10
7	27.0	497	-13,420	1,568	218	35.80	447	48.90
8	9.3	541	- 5,030	587	310	50.90	465	50.90
C	0	541	0	0	310	50.90	465	50.90

Horizontal Reaction A = 220#
Vertical Reaction A =-375#

position midway between points 5 and 6 on the upper rafter member.

The maximum horizontal shear is 42.6 #/sq.in. This is approximately one-third of the allowable working stress and is of no concern in the design of this roof.

The largest direct stress is 22.5 #/sq.in., which is also too low to be of any importance.

The maximum horizontal reaction is 233# at A. The largest vertical reaction is 124# at E. Due consideration should be given these reactions when the plate extends very far above the mow floor, as it is likely to cause too large stresses in the studding.

End wind. A graphical stress analysis of this barn roof is shown in Figure 17. Table 10 shows a summary of the moments, shears, and stresses determined in this analysis.

The maximum bending moment found is 21,350 in.lbs. at point 3. This bending moment results in a fiber stress of 2,495 #/sq.in. The range of the moments of any significance is approximately the same as when the wind is from the side, and the results indicate that the braces should be placed in the same position as for the side wind. The bending moment at the joints is 17,610 in. lbs.

The largest horizontal shear is 58.45 #/sq.in., which is too low to be a limiting factor in this design.

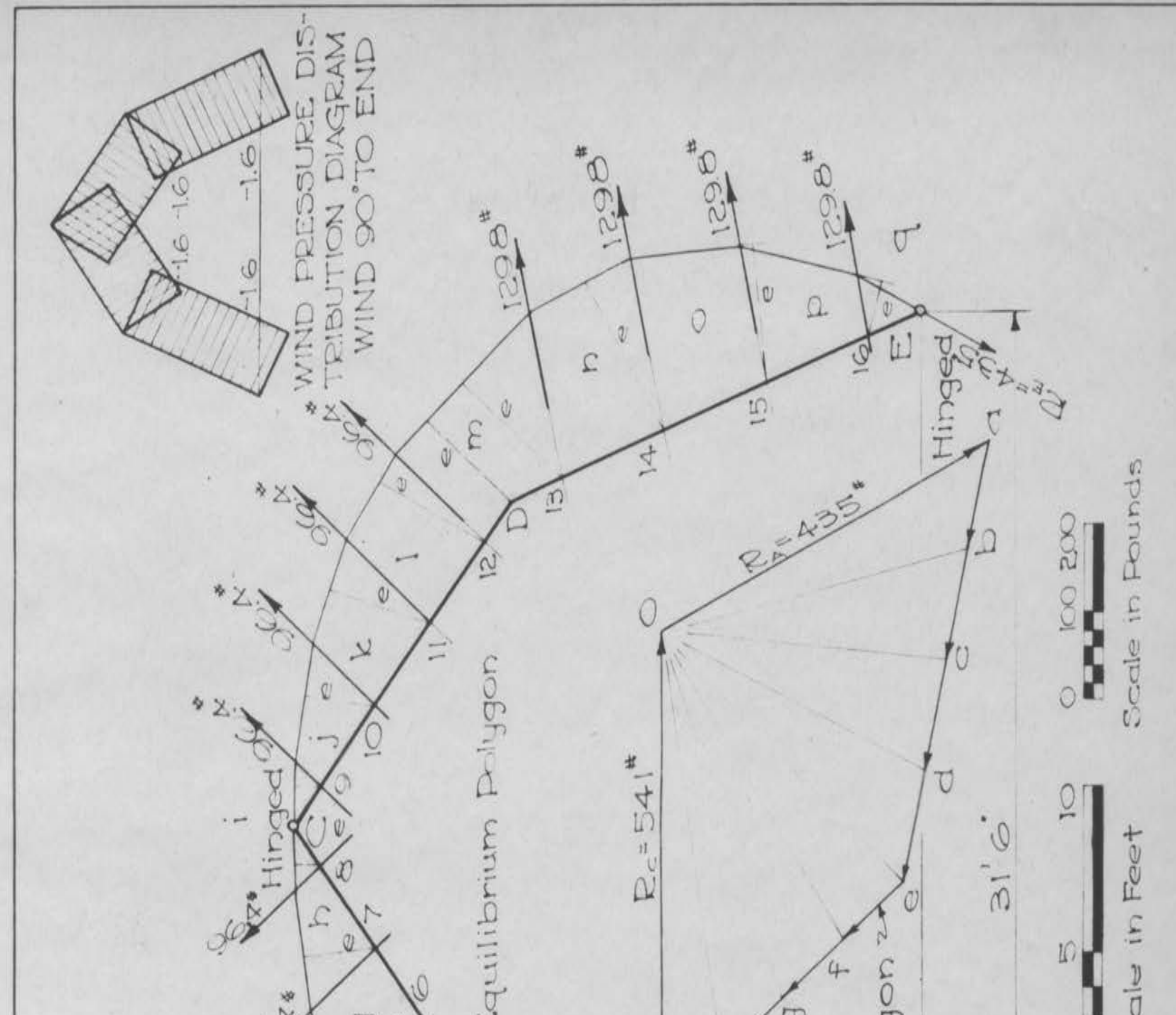


Fig. 17-Stress Analysis-Combined Dead and Wind Loads
32' width-2 rafter-End wind

The maximum direct stress is 50.9 #/sq.in., which is of no concern in this analysis.

The horizontal reaction at the plates is 220#. The vertical reaction is -375#. Both the vertical and horizontal reactions should be given due consideration in this design. The horizontal reaction may overstress the studding unless it is designed to resist moment caused by this reaction. The tendency of the barn roof with the wind to the end is to lift off of the plates. A toe nailed joint of at least four 16d nails would be required to resist the vertical pull of this roof. The reaction at the ridge is 541#, which is of considerable importance in designing the rafter tie at that point.

Combined dead and wind load stress analysis, 34' barn with 14' and 12' rafter members

The wind load data used in the stress analyses of the 34' barn roofs are shown in Table 11. The dead load and wind load are combined into resultants, which are considered as acting at four equally divided sections on each rafter member.

Side wind. A graphical stress analysis of this barn roof, with the wind to the side, is shown in Figure 18. Table 12 shows a summary of the moments, shears, and stresses determined in this analysis.

The maximum bending moment found is 15,550 in.lbs. at point 3 on the windward side. This moment creates a fiber

Table 11. Wind Load Data for Two Rafter Gambrel Barn Roofs
34' Barn - 70 M.P.H. Wind

Wind 90° to Side

			14' & 12' Rafter Members			16' & 12' Rafter Members		
Load No.	Coefficient	Pressure #/sq.'	Total P #	Dead Load #	Resultant #	Total P #	Dead Load #	Resultant #
1	1.0	12.55	88.0	35	109.0	100.4	40	123.2
2	.965	12.55	85.0	35	106.7	97.0	40	119.7
3	.815	12.55	71.7	35	93.7	81.9	40	105.4
4	.476	12.55	41.9	35	66.4	47.8	40	75.2
5	-.500	12.55	37.6	30	19.0	37.6	30	20.3
6	-.60	12.55	45.2	30	24.7	45.2	30	25.6
7	-.60	12.55	45.2	30	24.7	45.2	30	25.6
8	-.60	12.55	45.2	30	24.7	45.2	30	25.6
9	-.60	12.55	45.2	30	24.7	45.2	30	25.6
10	-.60	12.55	45.2	30	24.7	45.2	30	25.6
11	-.60	12.55	45.2	30	24.7	45.2	30	25.6
12	-.60	12.55	45.2	30	24.7	45.2	30	25.6
13	-.60	12.55	52.7	35	47.2	60.3	40	56.3
14	-.60	12.55	52.7	35	47.2	60.3	40	56.3
15	-.60	12.55	52.7	35	47.2	60.3	40	56.3
16	-.60	12.55	52.7	35	47.2	60.3	40	56.3

Wind 90° to End

1	-1.6	12.55	141.0	35	128.0	160.8	40	148.0
2	-1.6	12.55	141.0	35	128.0	160.8	40	148.0
3	-1.6	12.55	141.0	35	128.0	160.8	40	148.0
4	-1.6	12.55	141.0	35	128.0	160.8	40	148.0
5	-1.6	12.55	120.0	30	95.3	120.0	30	96.0
6	-1.6	12.55	120.0	30	95.3	120.0	30	96.0
7	-1.6	12.55	120.0	30	95.3	120.0	30	96.0
8	-1.6	12.55	120.0	30	95.3	120.0	30	96.0

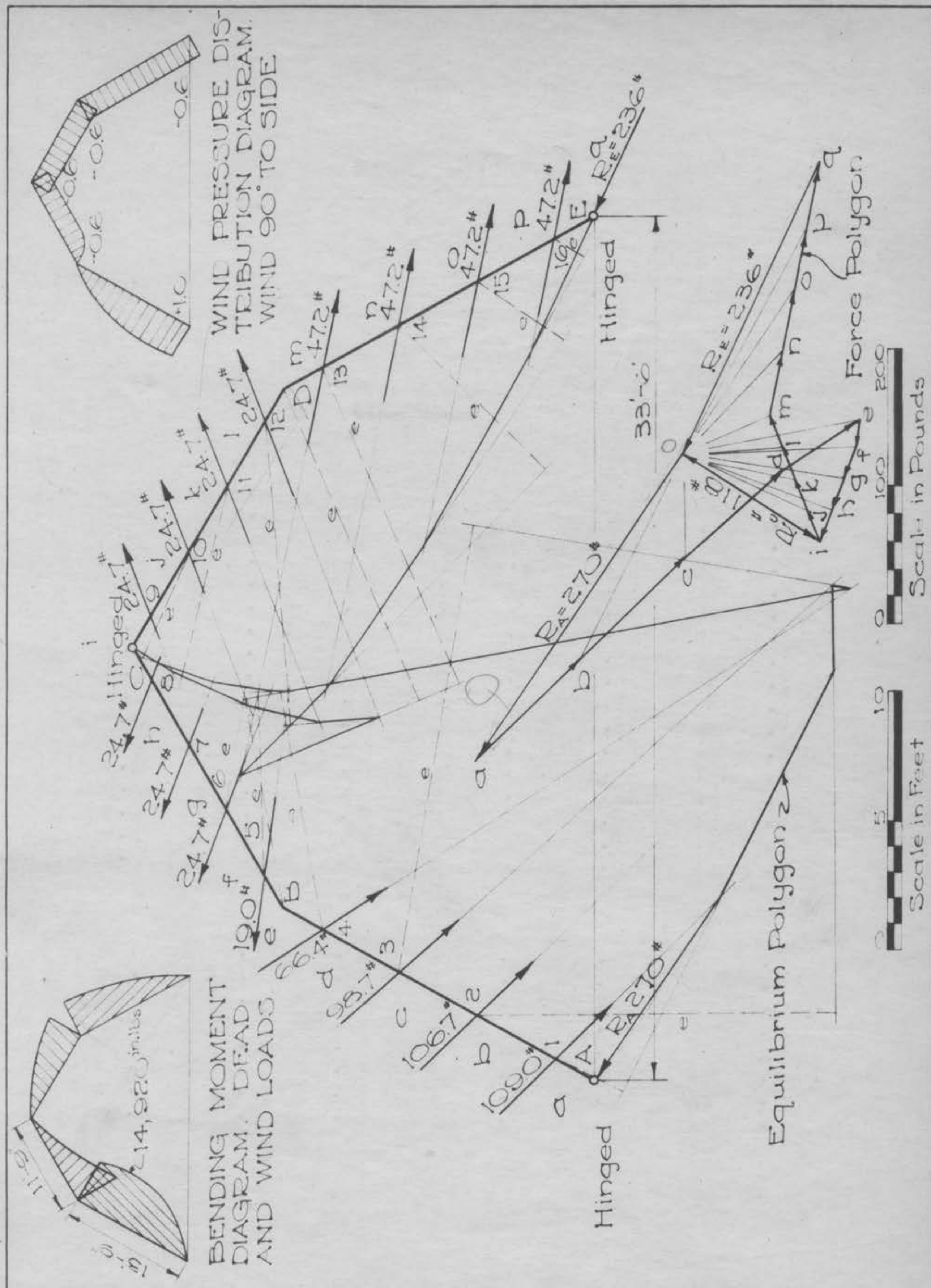


Fig.18-Stress Analysis-Combined Dead and Wind Loads
34' width - 2 rafter - Side wind

Table 12. Summary of Moments, Shears, and Stresses
Rafter Under Combined Dead and Wind Loads
34' Barn with 2"x6" x14' & 12' Rafter Members
Hinged at Plate and Ridge

Wind 90° to Side

Pt.	e in.	Thrust #	Moment in. #	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Stress #	Direct Fiber Stress #/sq."
A	0	270	0		268	44.05	21	2.99
1	34.0	166	+ 5,645	659	164	26.94	9	0.99
2	165.5	75	+12,420	1,451	64	10.51	39	4.27
3	210.0	74	+15,550	1,816	26	4.27	69	7.55
4	111.8	129	+14,420	1,683	83	13.62	101	11.70
B	98.0	129	+12,620	1,473	122	20.05	43	4.71
5	84.5	125	+10,570	1,232	110	18.07	58	6.35
6	57.8	118	+ 6,820	796	92	15.10	74	8.10
7	31.7	128	+ 4,060	474	72	11.82	89	9.74
8	8.5	118	+ 1,002	117	54	8.87	105	11.50
C	0	118	0	0	54	8.87	105	11.50
9	16.6	118	- 1,959	229	117	19.22	5	0.55
10	54.0	98	- 5,290	618	97	15.92	10	1.09
11	96.8	82	- 8,020	937	79	12.97	24	2.63
12	140.7	71	-10,000	1,168	59	9.69	40	4.38
D	158.0	68	-10,400	1,216	40	6.57	56	6.13
13	160.0	68	-10,880	1,272	6	0.99	67	7.33
14	94.8	103	- 9,760		30	4.93	100	10.94
15	48.0	147	- 7,060	825	66	10.83	130	14.23
16	15.3	190	- 2,907	340	102	16.75	161	17.62
E	0	236	0	0	139	22.85	193	21.12

Horizontal Reaction A = 224# Horizontal Reaction E = 216#
Vertical Reaction A = 149# Vertical Reaction E = 101#

Wind 90° to End

A	0	441	0		329	54.05	287	31.42
1	17.8	387	- 6,890	805	205	33.68	324	35.48
2	41.5	370	-15,360	1,792	81	13.30	357	39.07
3	47.7	396	-18,900	2,210	40	6.57	393	43.10
4	37.5	462	-17,320	2,025	162	26.61	428	46.85
B	30.0	462	-14,100	1,648	78	12.81	451	49.35
5	32.5	466	-15,150	1,770	15	2.46	467	51.15
6	29.5	492	-14,520	1,697	112	18.40	481	52.60
7	20.8	536	-11,150	1,303	203	33.35	495	54.20
8	7.3	589	- 4,310	503	300	49.28	510	55.85
C	0	589	0	0	300	49.28	510	55.85

Horizontal Reaction A = 149#
Vertical Reaction A =-413#

stress of 1,816 #/sq.in. This fiber stress is well under the maximum allowable fiber stress of 3,000 #/sq.in. As a result, the 2"x6" mmember can be assumed to carry the load without failure. The moment at the rafter joint B is 12,620 in. lbs. The bracing material on this rafter should extend from point 2 on the lower member to point 5 on the upper member.

The maximum horizontal shear in this rafter is 44.1 #/sq.in. The maximum direct fiber stress is 21.1#/sq.in. Neither of these stresses is large enough to take into consideration in the design of this roof.

The largest horizontal reaction is 224# at A. The largest vertical reaction is 149# at A. In considering the reactions, the horizontal reaction is the more critical. The studding must be checked for the stress that might be caused by this reaction. The reaction at the ridge is 118#.

End wind. A graphical stress analysis of this barn, with the end wind, is shown in Figure 19. Table 12 shows a summary of the moments, shears, and stresses determined in this analysis.

The maximum bending moment is -18,900 in. lbs. This moment causes a fiber stress of 2,210 #/sq.in. From the results obtained in this analysis, the bracing material should start relatively close to point 2 on the lower rafter and extend to point 5 on the upper rafter. The moment at the

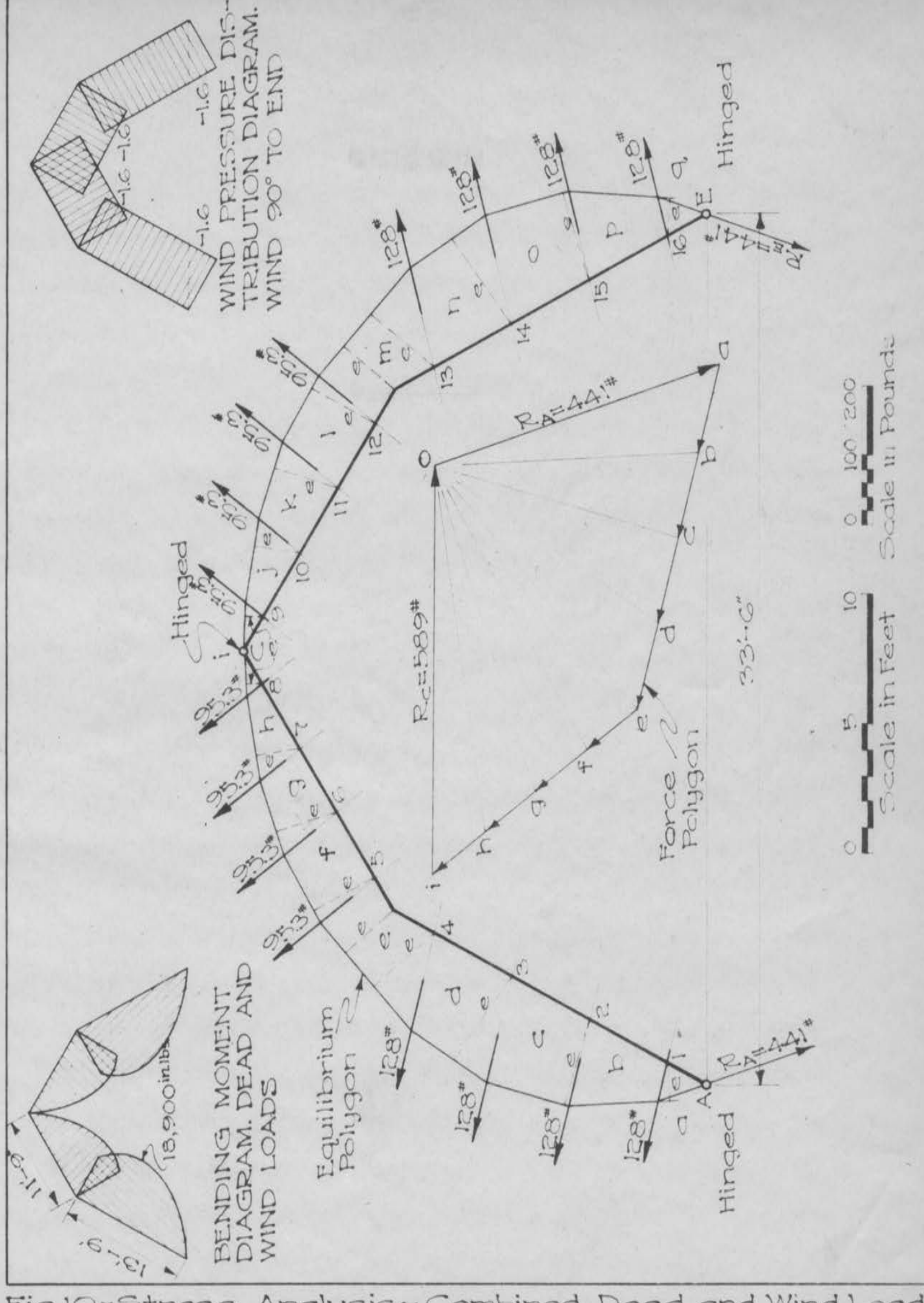


Fig.19-Stress Analysis-Combined Dead and Wind Loads.
34' width-2 rafter-End wind.

rafter joints is -14,100 in. lbs. The 2"x6" member will withstand these moments and stresses without failure.

The largest horizontal shear is 54.1 #/sq.in., and the largest direct fiber stress is 55.9 #/sq.in. These stresses are of no importance in this design.

The horizontal reaction at the plate is 149#. The vertical reaction at the same point is -413#. The reaction at C is 589#. A toe nailed joint, using four 16d nails, driven into a substantial plate member will safely resist the vertical pull of this rafter (27). This allows a factor of safety of about 5,8 for a 70 M.P.H. wind.

<u>Combined dead and wind load stress analysis, 34' barn with 16' and 12' rafter members</u>

<u>Side wind</u>. A graphical stress analysis of this barn, with a side wind, is shown in Figure 20. A summary of moments, shears, and stresses found in this analysis is presented in Table 13.

The maximum bending moment is 19,130 in. lbs. at point 3. This moment creates a fiber stress of 2,233 #/sq.in., which is considerably under the allowable stress of 3,000 #/sq.in. for this type of loading. The bending moment at the rafter joint is 15,620 in. lbs. It would be desirable in this design to have a brace from point 2 on the lower rafter member to point 5 on the upper rafter member. The bending

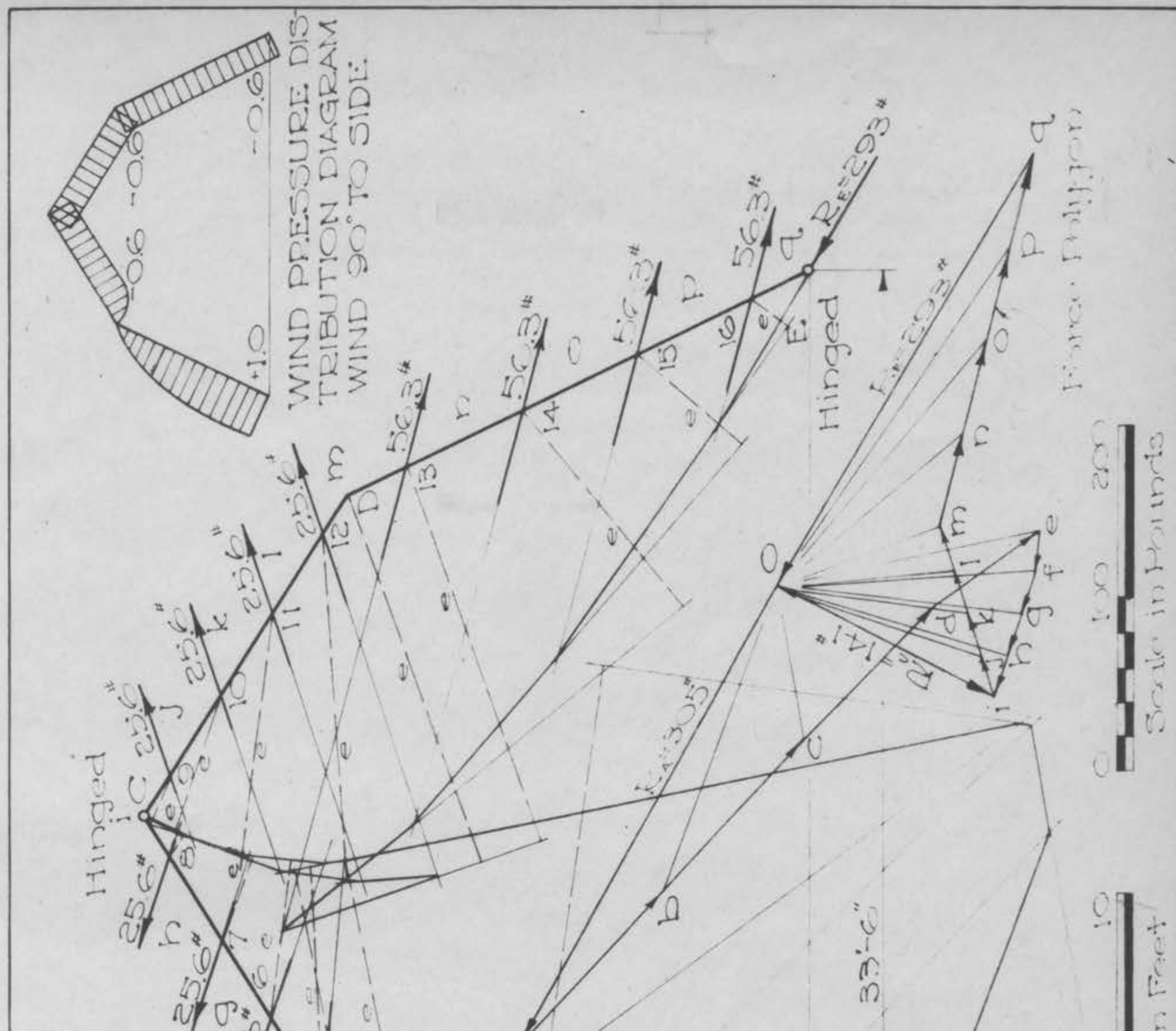


Fig. 20-Stress Analysis-Combined Dead and Wind Loads
34' width - 2 rafter - Side wind.

Table 13. Summary of Moments, Shears, and Stresses
Rafter Under Combined Dead and Wind Loads
34' Barn with 2"x6" x 16' & 12' Rafter Members
Hinged at Plate and Ridge

Wind 90° to Side

Pt.	e in.	Thrust #	Moment in. #	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Direct Stress #	Direct Fiber Stress #/sq."
A	0	305	0	0	304	49.90	20	2.19
1	37.2	187	+ 6,950	811	186	30.53	17	1.86
2	178.5	89	+15,890	1,854	73	11.98	51	5.58
3	208.0	92	+19,130	2,233	26	4.27	87	9.52
4	115.3	155	+17,900	2,090	93	15.25	124	13.58
B	100.8	155	+15,620	1,823	145	23.80	56	6.13
5	87.0	150	+13,050	1,523	132	21.68	73	7.99
6	58.6	143	+ 8,375	978	112	18.40	90	9.85
7	33.0	140	+ 4,620	539	91	14.93	106	11.60
8	8.8	141	+ 1,242	145	72	11.82	122	13.35
C	0	141	0	0	72	11.82	122	13.35
9	17.0	141	- 2,395	279	140	23.00	14	1.53
10	53.7	124	- 6,665	778	121	19.88	29	3.17
11	93.1	111	-10,330	1,208	101	16.59	46	5.03
12	129.2	102	-13,180	1,537	80	13.13	62	6.78
D	144.0	99	-14,250	1,664	60	9.85	78	8.53
13	146.6	99	-14,510	1,692	10	1.64	98	10.72
14	92.6	139	-12,880	1,503	32	5.26	135	14.77
15	49.5	187	- 9,260	1,080	76	12.48	170	18.60
16	15.5	239	- 3,705	432	119	19.54	206	22.55
E	0	293	0	0	162	26.60	244	26.70

Horizontal Reaction A = 266# Horizontal Reaction E = 251#
Vertical Reaction A = 150# Vertical Reaction E = 150#

Wind 90° to End

Pt.	e in.	Thrust #	Moment in. #	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Direct Stress #	Direct Fiber Stress #/sq."
A	0	465	0	0	385	63.22	262	28.65
1	23.5	382	- 8,985	1,050	241	39.60	299	32.75
2	58.1	350	-20,300	2,370	97	15.91	336	36.78
3	66.5	373	-24,800	2,898	47	7.72	372	40.70
4	50.5	449	-22,680	2,650	188	30.88	408	44.65
B	40.8	449	-18,330	2,145	55	9.04	446	48.80
5	41.6	463	-19,240	2,248	40	6.57	461	50.45
6	36.0	496	-17,830	2,085	134	22.10	480	52.60
7	24.0	542	-13,020	1,520	229	37.60	494	53.10
8	8.6	603	- 5,185	606	323	53.10	510	55.85
C	0	603	0	0	323	53.10	510	55.85

Horizontal Reaction A = 233#
Vertical Reaction A =-404#

moment, created on the leeward side of this barn roof and on the same side of the other barn roofs, is considerably less than the bending moment on the windward side. In this analysis the maximum bending moment on the leeward side is -14,510 in.lbs. at point 13 on the lower rafter member.

The greatest horizontal shear is 49.9 #/sq.in., and the greatest direct fiber stress is 26.7 #/sq.in. Neither of these stresses is a limiting factor in the design of this barn roof.

The greatest horizontal reaction at the plates is 266# at hinge A. The vertical reactions are 150# at both hinge A and E. The reaction at the ridge is 141#. The horizontal reaction at A is large enough that the stresses in the studding should be checked for this roof. A toe nailed joint of three 16d nails will safely resist the horizontal thrust at joint A.

End wind. A graphic stress analysis of this barn with an end wind is shown in Figure 21. A summary of the moments, shears, and stresses found in this analysis is presented in Table 13.

The maximum bending moment is -24,800 in.lbs. at point 3. This bending moment creates a fiber stress of 2,898 #/sq. in., which is below the allowable working stress of 3,000 #/sq. in.

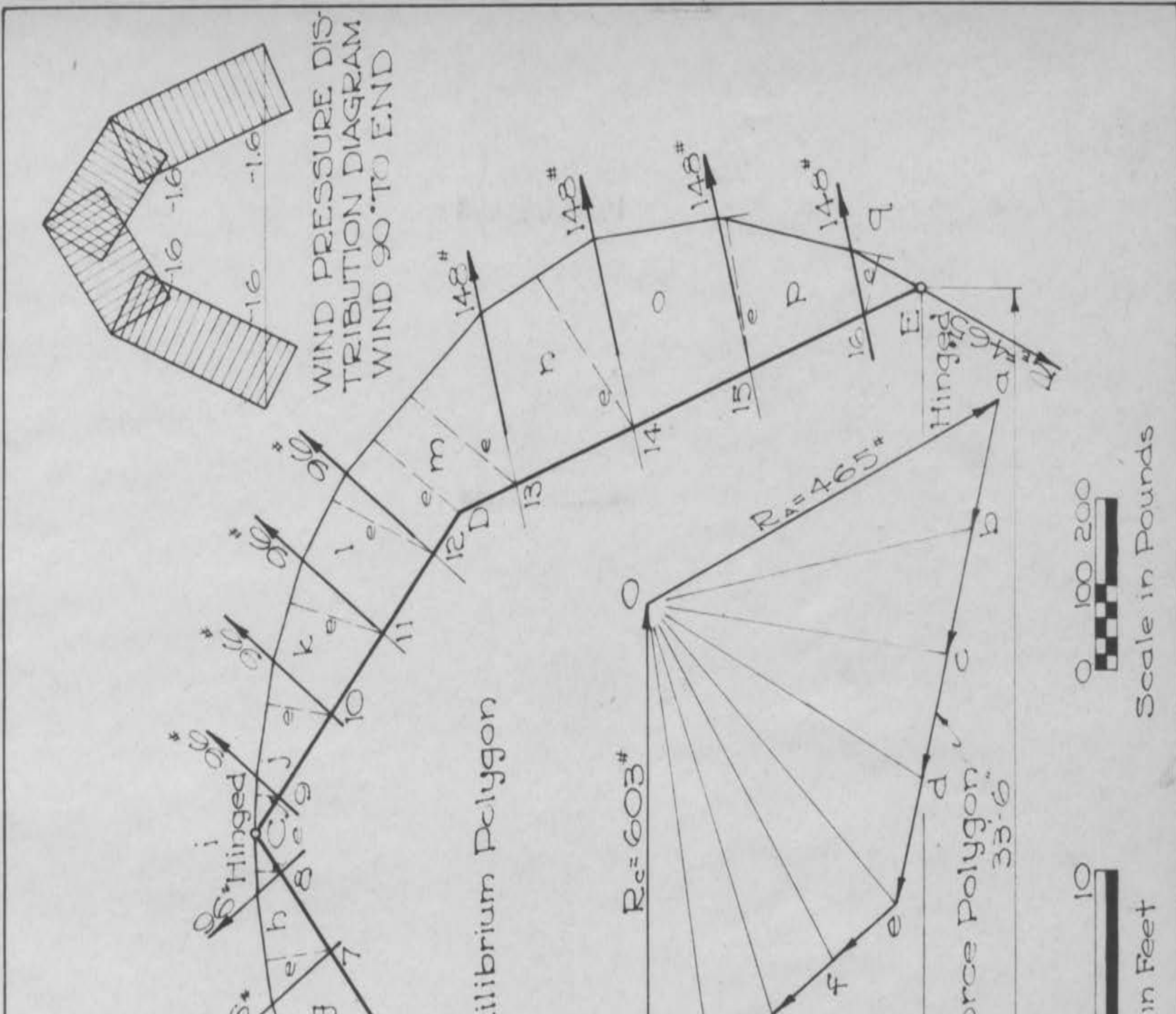


Fig. 21-Stress Analysis-Combined Dead and Wind Loads
34' width - 2 rafter - End wind

The results of this investigation show that the rafter bracing will be much more valuable if it is placed between point 2 on the lower rafter member and point 5 on the upper rafter member. The bending moment at the joints is -18,330 in. lbs.

The maximum horizontal shear found in this roof analysis is 63.2 #/sq.in. The maximum direct fiber stress is 55.9 #/sq.in. These stresses are too low to be of any importance in the design of this roof.

The horizontal thrust at the plates is 233#. The vertical reaction is -404#. The reaction at the ridge is 603#. These reactions are important factors in selecting methods of fastening the rafters at the ridge and plates. A toe nailed joint of at least four 16d nails would be required to resist the vertical pull of this roof. However, toe nailed joints should not be depended upon alone to hold a rafter at the plate. It is desirable to have a rafter brace or tie extending from the rafter member to the stud.

<u>Combined dead and wind load stress analysis, 36' barn with 16' and 12' rafter members.</u>

The wind load data used in the stress analyses of the 36' barn roofs is shown in Table 14. The dead load and wind load are combined into resultants, which are considered as acting at four equally divided sections on each rafter

Table 14. Wind Load Data for Two Rafter Gambrel Barn Roofs
36' Barn - 70 M.P.H. Wind

Wind 90° to Side

			16' & 12' Rafter Members			16' & 14' Rafter Members		
Load No.	Coefficient	Pressure #/sq.'	Total P #	Dead Load #	Resultant #	Total P #	Dead Load #	Resultant #
1	1.0	12.55	100.4	40	127.5	100.4	40	122.7
2	0.965	12.55	96.8	40	120.0	96.8	40	118.9
3	0.815	12.55	81.8	40	107.0	81.8	40	105.2
4	0.476	12.55	47.7	40	75.3	47.7	40	73.8
5	-0.50	12.55	37.6	30	18.7	47.1	35	27.7
6	-0.6	12.55	45.2	30	24.0	52.7	35	31.5
7	-0.6	12.55	45.2	30	24.0	52.7	35	31.5
8	-0.6	12.55	45.2	30	24.0	52.7	35	31.5
9	-0.6	12.55	45.2	30	24.0	52.7	35	31.5
10	-0.6	12.55	45.2	30	24.0	52.7	35	31.5
11	-0.6	12.55	45.2	30	24.0	52.7	35	31.5
12	-0.6	12.55	45.2	30	24.0	52.7	35	31.5
13	-0.6	12.55	60.3	40	54.3	60.3	40	56.7
14	-0.6	12.55	60.3	40	54.3	60.3	40	56.7
15	-0.6	12.55	60.3	40	54.3	60.3	40	56.7
16	-0.6	12.55	60.3	40	54.3	60.3	40	56.7

Wind 90° to End

1	-1.6	12.55	161.0	40	146.0	161.0	40	148.7
2	-1.6	12.55	161.0	40	146.0	161.0	40	148.7
3	-1.6	12.55	161.0	40	146.0	161.0	40	148.7
4	-1.6	12.55	161.0	40	146.0	161.0	40	148.7
5	-1.6	12.55	120.0	30	95.0	141.0	35	114.2
6	-1.6	12.55	120.0	30	95.0	141.0	35	114.2
7	-1.6	12.55	120.0	30	95.0	141.0	35	114.2
8	-1.6	12.55	120.0	30	95.0	141.0	35	114.2

member.

<u>Side</u> <u>wind</u>. A graphical stress analysis of this barn roof, with the wind to the side, is shown in Figure 22. A summary of the moments, shears, and stresses determined in this analysis is presented in Table 15.

The maximum bending moment found is 19,780 in. lbs. at point 3 on the lower rafter member. This moment creates a fiber stress of 2,308 #/sq.in., which is considerably under the allowable working stress of 3,000 #/sq.in. The bending moment at the rafter joint is 16,120 in. lbs. on the windward side. The moments on the leeward side are smaller than the ones on the windward side; therefore, it is not necessary to consider them in this design. The rafter brace on this barn should be located between point 2 on the lower rafter member and point 5 on the upper rafter member.

The maximum horizontal shear is 50.7 #/sq.in., and the maximum direct fiber stress is 24.9 #/sq.in. It is not necessary to consider either of these stresses in this design.

The largest reaction found in this analysis is 310# at plate A. The horizontal component of this reaction is 262#. The vertical component is 166#. The reaction at C is 132#. The stresses created in the studding due to the horizontal reaction should be checked for this roof. The rafter member for this roof should be connected to the plate with at least

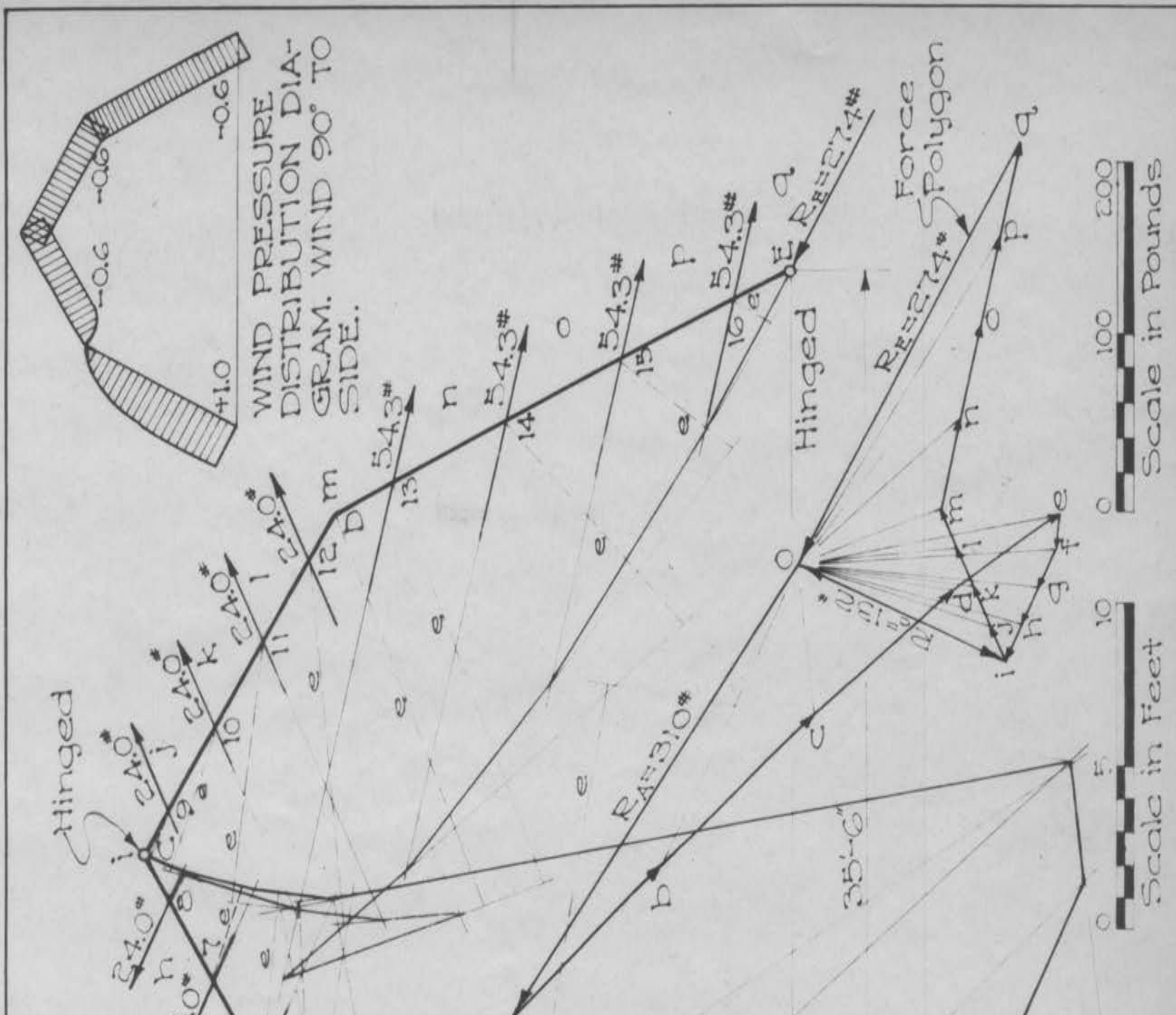


Fig.22-Stress Analysis-Combined Dead and Wind Loads.
36' width-2 rafter-Side wind.

Table 15. Summary of Moments, Shears, and Stresses
Rafter Under Combined Dead and Wind Loads
36' Barn with 2"x6" x 16' & 12' Rafter Members
Hinged at Plate and Ridge

Wind 90° to Side

Pt.	e in.	Thrust #	Moment in. #	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Direct Stress #	Direct Fiber Stress #/sq."
A	0	310	0	0	309	50.74	24	2.63
1	38.2	187	+ 7,140	834	186	30.54	12	1.31
2	186.2	87	+16,210	1,892	72	11.82	48	5.26
3	222.2	89	+19,780	2,308	28	4.60	84	9.19
4	120.0	153	+18,360	2,143	95	15.60	121	13.24
B	105.3	153	+16,120	1,880	145	23.80	47	5.14
5	91.7	148	+13,580	1,584	133	21.85	64	7.00
6	63.5	138	+ 8,770	1,024	114	18.72	79	8.64
7	35.0	133	+ 4,660	544	95	15.60	92	10.08
8	10.5	132	+ 1,385	1,617	76	12.47	107	11.72
C	0	132	0	0	76	12.47	107	11.72
9	17.5	132	- 2,310	270	131	21.50	9	0.99
10	54.7	116	- 6,350	741	114	18.72	23	2.52
11	95.0	102	- 9,690	1,132	93	15.27	38	4.16
12	133.6	92	-12,290	1,435	75	12.32	52	5.69
D	152.5	87	-13,270	1,549	56	9.19	67	7.33
13	155.2	87	-13,505	1,577	11	1.81	88	9.63
14	98.0	125	-12,250	1,430	30	4.93	123	13.47
15	50.7	172	- 8,730	1,019	71	11.65	158	17.39
16	15.5	222	- 3,440	401	112	18.38	193	21.13
E	0	274	0	0	154	25.28	228	24.95

Horizontal Reaction A = 262# Horizontal Reaction E = 242#
Vertical Reaction A = 166# Vertical Reaction E = 128#

Wind 90° to End

Pt.	e in.	Thrust #	Moment in. #	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Direct Stress #	Direct Fiber Stress #/sq."
A	0	475	0	0	370	60.72	299	32.72
1	21.5	405	- 8,710	1,018	228	37.42	333	36.44
2	51.5	380	-19,580	2,285	88	14.43	369	40.37
3	57.8	411	-23,750	2,713	54	8.86	406	44.40
4	43.0	482	-20,730	2,420	195	32.03	441	48.20
B	33.5	482	-16,160	1,886	75	12.32	478	52.30
5	36.0	489	-17,620	2,058	19	3.12	494	54.10
6	32.6	523	-17,050	1,990	113	18.54	508	55.60
7	22.7	566	-12,830	1,490	210	34.48	523	57.20
8	8.9	617	- 5,490	641	301	49.40	537	59.90
C	0	617	0	0	301	49.40	537	59.90

Horizontal Reaction A = 111#
Vertical Reaction A =-437#

three 16d nails. There should also be a rafter tie from the end of the lower rafter member to the studding.

End wind. A graphical stress analysis of this barn, with an end wind, is shown in Figure 23. A summary of the moments, shears, and stresses found in this analysis is shown in Table 15.

The maximum bending moment is -23,750 in. lbs. at point 3 on the lower rafter member. This moment produces a fiber stress of 2,713 #/sq.in. This stress is far enough under the allowable of 3,000 #/sq.in. to be considered safe. The bending moment at the joints is -16,160 in. lbs. The brace on this rafter should extend froma point midway between 1 and 2 on the lower rafter member to a point midway between point B and point 5 on the upper rafter member.

The maximum horizontal shear is 60.7 #/sq.in. and the maximum direct fiber stress is 59.9 #/sq.in. These stresses are not large enough to consider in this design.

The reaction at the plate is 475#. The horizontal component of this reaction is 111#; the vertical component is -417#. The reaction at the ridge is 617#. Each of these reactions is large enough to warrant considerable attention in fastening the rafters for this barn roof.

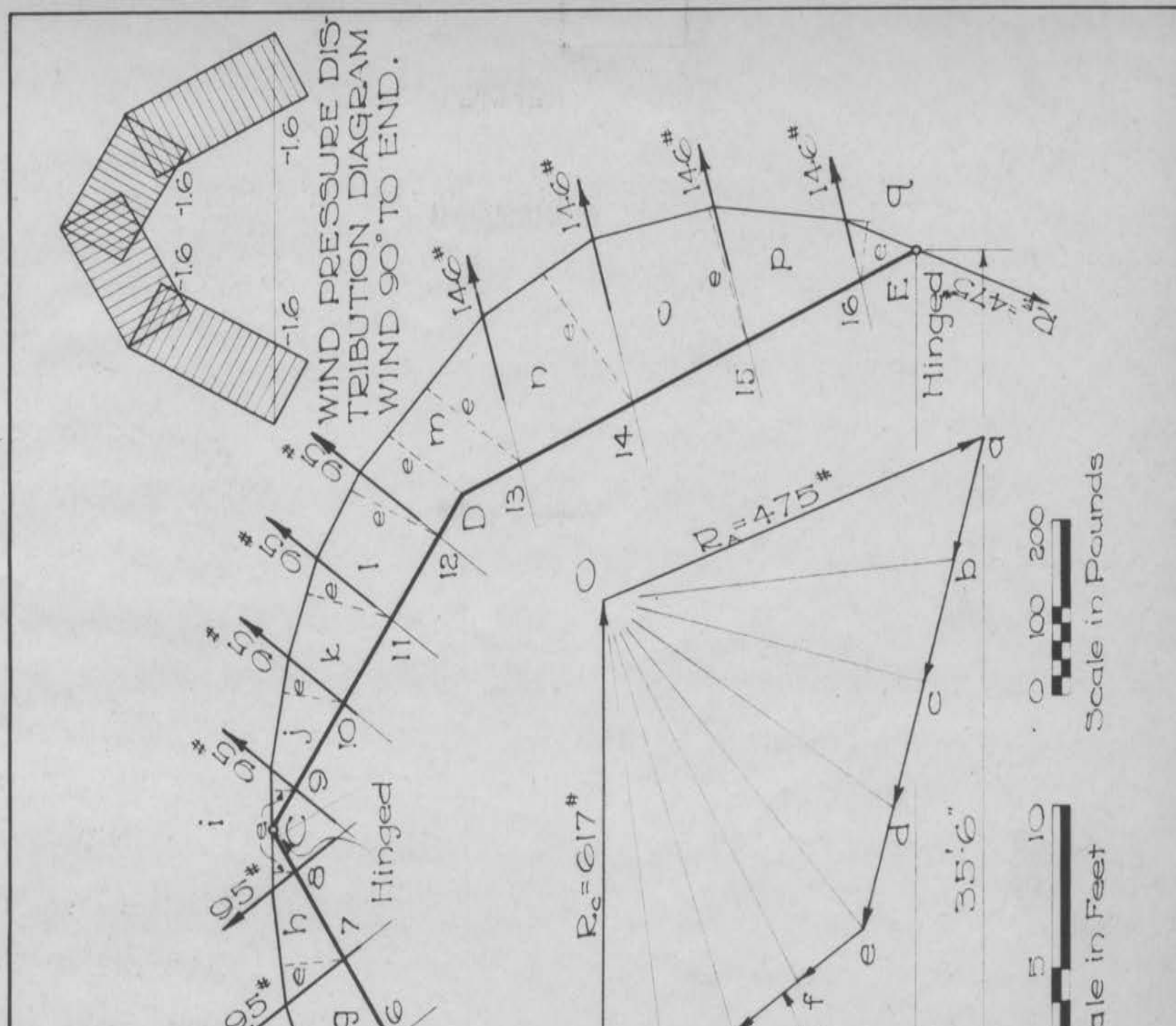


Fig.23-Stress Analysis-Combined Dead and Wind Loads
36' width-2 rafter-End wind.

Combined dead and wind load stress analysis, 36' barn with 16' and 14' rafter members

Side wind. Figure 24 shows a graphic stress analysis of this barn roof with the wind to the side. A summary of the moments, shears, and stresses is presented in Table 16.

The maximum bending moment, determined in the analysis of this roof, is 19,200 in. lbs. at point 3 on the lower rafter member. The fiber stress created by this moment is 2,242 #/ sq. in. This rafter will take care of the load produced by the 70 M.P.H. wind without failure. The maximum bending moment at the joints is 15,640 in. lbs. at joint B. With this joint braced with material equally as good as the rafter, no failure should result here.

The maximum horizontal shear is 49.4 # /sq.in. The maximum direct fiber stress is 21.1 #/sq.in. No failure would be expected to result from these stresses.

The largest reaction is 303 # at plate A. The horizontal component of this reaction is 262#. The vertical component is 151#. The reaction at the ridge is 144#. In the large barns, such as the 36' barn and 40' barn, it becomes increasingly important to give more consideration to the reactions at the plate and ridge.

End wind. A graphical stress analysis of this barn roof, with the wind to the end, is shown in Figure 25. Table 16

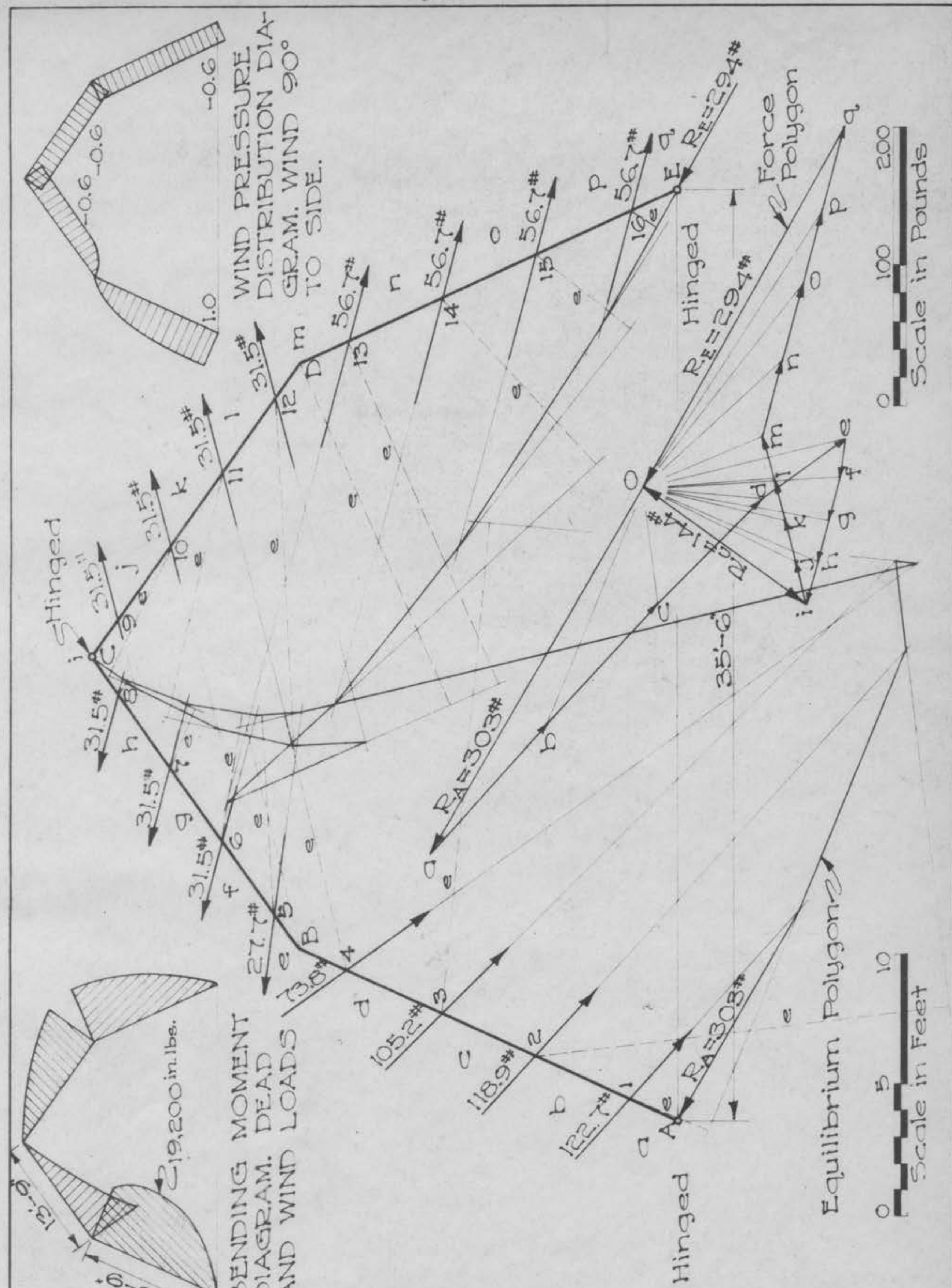


Fig. 24- Stress Analysis- Combined Dead and Wind Loads.
36' width- 2 rafter- Side wind.

Table 16. Summary of Moments, Shears, and Stresses
Rafter Under Combined Dead and Wind Loads
36' Barn with 2"x6" x 16' & 14' Rafter Members
Hinged at Plate and Ridge

Wind 90° to Side

Pt.	e in.	Thrust #	Moment in. #	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Direct Stress #	Direct Fiber Stress #/sq."
A	0	303	0	0	301	49.40	29	3.18
1	38.8	184	+ 7,140	834	184	30.20	7	0.77
2	191.0	83	+15,840	1,850	72	11.82	43	4.71
3	225.8	85	+19,200	2,242	26	4.27	81	8.87
4	120.0	149	+17,880	2,088	91	14.95	118	12.92
B	105.1	149	+15,640	1,828	138	22.65	55	5.47
5	91.3	141	+12,880	1,505	120	19.70	76	8.32
6	58.5	135	+ 7,900	923	95	15.60	96	10.51
7	28.8	136	+ 3,915	457	72	11.82	116	12.70
8	7.0	144	+ 1,008	118	47	7.72	136	14.90
C	0	144	0	0	47	7.72	136	14.90
9	23.3	144	- 3,355	391	144	23.65	0	0
10	65.2	122	- 7,950	929	120	19.71	20	2.19
11	114.5	104	-11,910	1,392	96	15.76	40	4.38
12	159.2	94	-14,980	1,750	72	11.82	60	6.57
D	170.8	94	-16,060	1,876	48	7.89	80	8.76
13	171.4	94	-16,120	1,883	2	0.33	93	10.18
14	103.4	136	-14,080	1,645	41	6.74	130	14.23
15	54.0	147	- 7,940	928	85	13.97	165	18.07
16	16.4	240	- 3,938	460	130	21.35	201	22.00
E	0	294	0	0	174	28.60	238	26.05

Horizontal Reaction A = 262# Horizontal Reaction E = 257#
Vertical Reaction A = 151# Vertical Reaction E = 144#

Wind 90° to End

Pt.	e in.	Thrust #	Moment in. #	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Direct Stress #	Direct Fiber Stress #/sq."
A	0	505	0	0	418	68.70	281	30.78
1	23.7	420	- 9,960	1,164	274	45.00	317	34.70
2	60.8	376	-22,860	2,670	129	21.18	352	38.55
3	75.0	389	-29,200	3,410	13	0.21	389	42.60
4	62.5	453	-28,320	3,310	158	25.97	424	46.40
B	54.2	453	-24,550	2,870	75	12.32	447	49.00
5	55.7	468	-26,080	3,038	37	6.07	467	51.15
6	48.0	508	-24,400	2,850	150	24.63	487	53.30
7	32.2	570	-18,350	2,143	262	43.10	507	55.50
8	12.5	645	- 8,060	941	375	61.60	527	57.70
C	0	645	0	0	375	61.60	527	57.70

Horizontal Reaction A = 267#
Vertical Reaction A =-430#

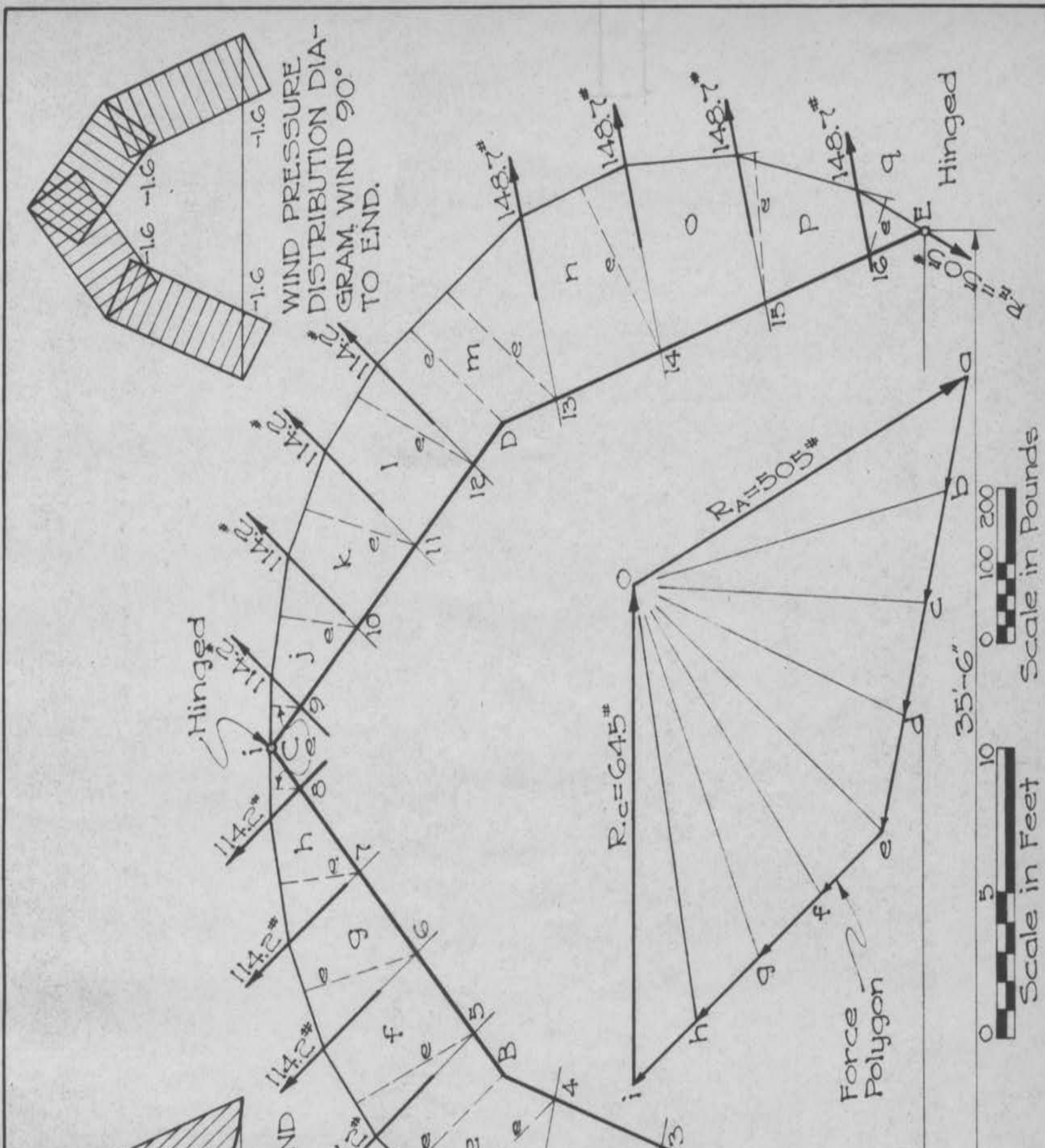


Fig.25-Stress Analysis-Combined Dead and Wind Loads.
36' width-2 rafter-End wind.

shows a summary of the moments, shears, and stresses found in this analysis.

The maximum bending moment created by this load is -29,200 in. lbs. at point 3 on the lower rafter member. This bending moment creates a fiber stress of 3,410 #/sq.in. This fiber stress is too large for the 2"x6" member. As it is necessary to supply a rafter brace or tie to connect the rafter members, it may be possible to place this brace in such a position that it will strengthen the rafter members where they are overstressed. The rafter brace for this barn should extend from a position midway between point 2 and 3 on the lower rafter to midway between point 5 and 6 on the upper rafter member. The bending moment at the joints is -24,550 in. lbs. This moment results in a fiber stress of only 2,820 #/sq.in.; therefore, it is not necessary to use material with a larger moment of inertia about its neutral axis than the moment of inertia of a 2"x6" member about its neutral axis.

The maximum horizontal shear in this rafter is 68.7 #/sq. in., and the maximum direct stress is 57.7 #/sq.in. These stresses are not important in the design of this roof.

The reaction at the plate, in this analysis, is 505#. The horizontal component of this reaction is 267#. The vertical component is -430#. The reaction at the ridge of this barn roof is 645#. Careful consideration should be given to connecting the rafter of this barn to the plate and to the connection at the ridge.

Stress analyses of three rafter gambrel barn roofs

The method used in making stress analyses of the three rafter gambrel barn roofs is the same as that used in analyzing the two rafter gambrel barn roofs. This method is summarized on page 58.

Dead load stress analyses of the 36' and 40' barn roofs

36' barn with 14',10' & 8' rafters. A graphic stress analysis of this barn under dead load is presented in Figure 26. Table 17 shows a summary of the moments, shears, and stresses taken from this analysis. The moments in this analysis were calculated algebraically and checked graphically.

The maximum bending moment found is 1,075 in. lbs. at points 2 and 3. This moment produces a fiber stress of 125.5 #/sq.in., which is relatively small when compared to the allowable stress of 1,200 #/sq.in. for permanent loading. No moment is created at the rafter joints as a stable roof shape was used in this analysis.

The maximum horizontal shear is 6.24 #/sq.in. and the maximum direct stress is 36.6 #/sq.in. These stresses are of little concern when a stable roof shape is used.

The reaction at A is 335#. The horizontal component of

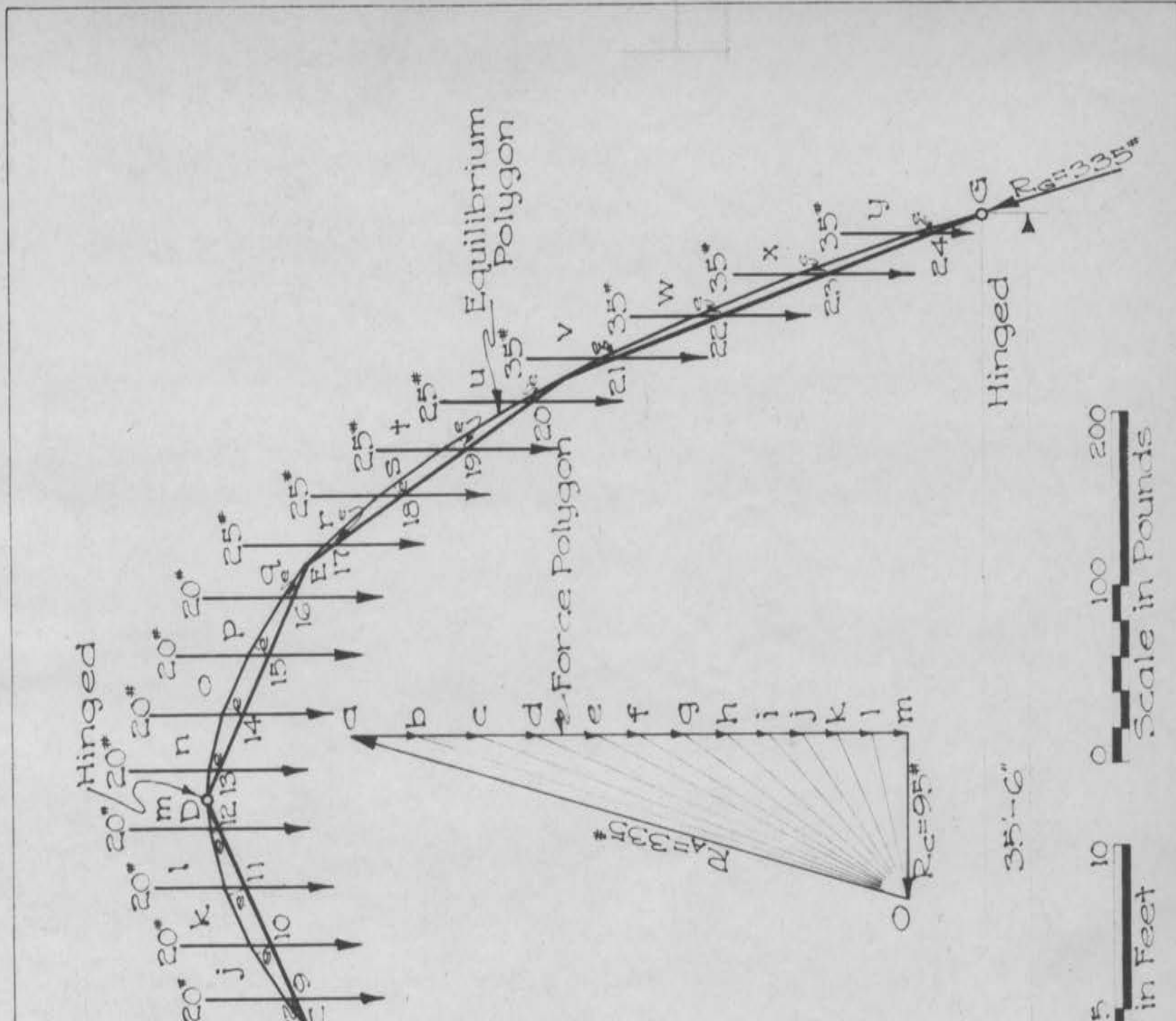


Fig. 26. Stress Analysis - Dead Load.
36' width-3 rafter--Stable shape.

Table 17. Summary of Moments, Shears, and Stresses
Rafter Under Dead Load Hinged at Plate and Ridge
36' Barn with 2"x6" x 14', 10' & 8' Rafter Members

Pt.	e in.	Thrust #	Moment in. #	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Direct Stress #	Direct Fiber Stress #/sq."
A	0.00	355	0	0	27	4.44	334	36.58
1	1.61	335	+ 541	63.1	27	4.44	331	36.58
2	3.56	302	+1,075	125.5	14	2.30	301	32.95
3	4.00	269	+1,075	125.5	0	0.00	269	29.45
4	2.28	237	+ 541	63.1	12	1.97	236	25.85
B	0.00	204	0	0.0	24	3.94	202	22.12
5	2.13	204	+ 435	50.8	30	4.93	202	22.12
6	4.24	182	+ 772	90.1	14	2.30	181	19.82
7	4.76	162	+ 772	90.1	0	0.00	162	17.73
8	3.07	142	+ 435	50.8	15	2.46	141	15.43
C	0.00	126	0	0.0	29	4.76	122	13.36
9	3.92	126	+ 494	57.6	36	5.81	120	13.13
10	7.88	114	+ 898	104.9	17	2.79	112	12.27
11	8.64	104	+ 898	104.9	0	0.00	104	11.39
12	5.04	98	+ 494	57.6	18	2.96	96	10.51
D	0.00	95	0	0.0	38	6.24	87	9.52

Horizontal Reaction A = 95# Vertical Reaction A = 320#

Table 18. Summary of Moments, Shears, and Stresses
Rafter Under Dead Load Hinged at Plate and Ridge
40' Barn with 2"x6" x 14', 12', & 8' Rafter Members

A	0.00	358	0	0.0	26	4.27	356	38.95
1	1.52	358	+ 550	63.2	26	4.27	356	38.95
2	3.37	325	+1,096	128.0	13	2.14	324	35.45
3	3.75	292	+1,096	128.0	0	0.00	292	31.95
4	2.11	260	+ 550	63.2	13	2.14	259	28.38
B	0.00	228	0	0.0	27	4.44	227	24.85
5	2.59	228	+ 590	68.9	39	6.41	226	24.75
6	5.55	203	+1,128	131.8	20	3.29	202	22.13
7	6.33	178	+1,128	131.8	0	0.00	178	19.50
8	3.78	156	+ 590	68.9	18	2.96	155	16.97
C	0.00	136	0	0.0	36	5.81	131	14.32
9	3.23	136	+ 440	51.4	36	5.81	131	14.32
10	7.04	125	+ 880	102.8	19	3.12	124	13.58
11	7.52	117	+ 880	102.8	0	0.00	117	12.80
12	3.93	112	+ 440	51.4	18	2.96	110	12.05
D	0.00	110	0	0.0	38	6.24	103	11.28

Horizontal Reaction A = 110# Vertical Reaction A = 340#

this reaction is 95#. The vertical component is 320#, which is the weight of the roof for that side.

40' barn with 14', 12', & 8' rafters. A graphic stress analysis of this barn under dead load is presented in Figure 27. Table 18 shows a summary of the moments, shears, and stresses taken from this analysis. The moments in this analysis were calculated algebraically and checked graphically.

The maximum bending moment found is 1,128 in.lbs. at point 6 on the middle rafter member. The stress created by this moment is 131.8 #/sq.in. This stress is not likely to cause any large amount of deflection over a long period of time.

The maximum horizontal shear is 6.24 #/sq.in., and the maximum direct fiber stress is 39 #/sq.in. These stresses are of little concern in a roof subjected to dead loads.

The reaction at A is 358#. The horizontal component of this reaction is 110#; the vertical component is 340#, which is the weight of the roof on that side.

Combined dead and wind load stress analysis, 36' barn with 14',10', and 8' rafter members

The wind load data used in the stress analyses of the 36' three rafter gambrel barn roofs are shown in Table 19. The dead load and wind load are combined into resultants, which are considered as acting at four equally divided

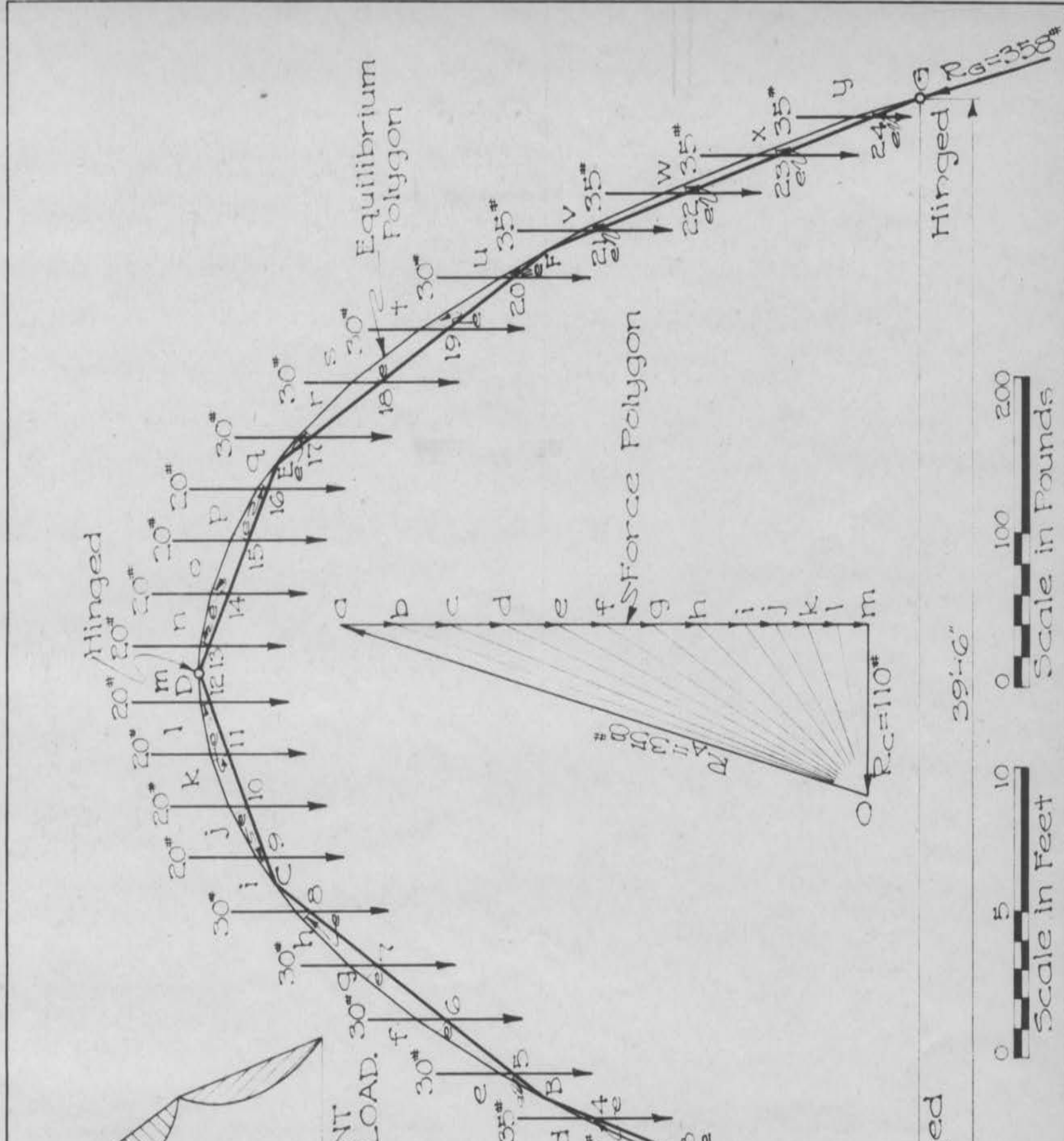


Fig.27-Stress Analysis- Dead Load
40' width-3 rafter-Stable shape.

Table 19. Wind Load Data for Three Rafter Gambrel Barn Roofs
36' Barn - 70 M.P.H. Wind

Wind 90° to Side

			14', 10', & 8' Rafter Members			14', 12', & 8' Rafter Members		
Load No.	Coefficient	Pressure #/sq.'	Total P #	Dead Load #	Resultant #	Total P #	Dead Load #	Resultant #
1	0.987	12.55	86.7	35	104.3	86.7	35	103.7
2	0.963	12.55	84.7	35	102.4	84.7	35	101.5
3	0.937	12.55	82.3	35	100.7	82.3	35	100.5
4	0.913	12.55	80.3	35	98.5	80.3	35	97.6
5	0.870	12.55	54.6	25	72.3	65.5	30	86.1
6	0.745	12.55	46.7	25	64.5	56.1	30	76.8
7	0.555	12.55	34.8	25	53.3	41.8	30	63.6
8	0.230	12.55	14.4	25	35.4	17.3	30	42.3
9	-0.265	12.55	13.3	20	9.4	13.3	20	9.3
10	-0.596	12.55	29.9	20	14.1	29.9	20	14.6
11	-0.675	12.55	33.9	20	17.6	33.9	20	17.7
12	-0.695	12.55	34.9	20	18.5	34.9	20	18.8
13	-0.70	12.55	35.2	20	18.7	35.2	20	19.2
14	-0.70	12.55	35.2	20	18.7	35.2	20	19.2
15	-0.70	12.55	35.2	20	18.7	35.2	20	19.2
16	-0.70	12.55	35.2	20	18.7	35.2	20	19.2
17	-0.70	12.55	43.9	25	35.5	52.8	30	44.3
18	-0.70	12.55	43.9	25	35.5	52.8	30	44.3
19	-0.70	12.55	43.9	25	35.5	52.8	30	44.3
20	-0.70	12.55	43.9	25	35.5	52.8	30	44.3
21	-0.713	12.55	62.7	35	59.7	62.7	35	61.1
22	-0.737	12.55	64.7	35	61.5	64.7	35	62.7
23	-0.763	12.55	67.1	35	63.6	67.1	35	64.9
24	-0.787	12.55	69.2	35	65.5	69.1	35	66.8

Wind 90° to End

1	-1.6	12.55	141.0	35	132.8	141.0	35	133.3
2	-1.6	12.55	141.0	35	132.8	141.0	35	133.3
3	-1.6	12.55	141.0	35	132.8	141.0	35	133.3
4	-1.6	12.55	141.0	35	132.8	141.0	35	133.3
5	-1.6	12.55	100.5	25	88.2	120.0	30	106.8
6	-1.6	12.55	100.5	25	88.2	120.0	30	106.8
7	-1.6	12.55	100.5	25	88.2	120.0	30	106.8
8	-1.6	12.55	100.5	25	88.2	120.0	30	106.8
9	-1.6	12.55	80.3	20	62.5	80.3	20	58.2
10	-1.6	12.55	80.3	20	62.5	80.3	20	58.2
11	-1.6	12.55	80.3	20	62.5	80.3	20	58.2
12	-1.6	12.55	80.3	20	62.5	80.3	20	58.2

sections on each of the three rafter members.

Side wind. A graphic analysis of this barn roof is illustrated in Figure 28. Table 20 shows a summary of the moments, shears, and stresses taken from this diagram.

The maximum bending moment is 39,460 in. lbs. at joint B. This moment creates a fiber stress of 4,610 #/sq.in., which is considerably above the allowable of 3,000 #/sq.in. To reduce the stress below the allowable stress, it is necessary to use another member other than the 2"x6" member. By using a 2"x8" member, the fiber stress is reduced to 2,590#/sq.in., which is a safe stress for this loading.

The horizontal shear for this rafter is 71#/sq.in.; the direct fiber stress is 45.7 #/sq.in. The horizontal shear is the only one that could possibly be of any concern in this design. However, it is below the allowable shear of 120 #/sq.in.

The largest reaction at the plate for this roof is 509# at plate G. The horizontal component of this reaction is 420#; the vertical component is 287#. Since the three rafter gambrel barn roofs are fastened at the mow floor, these reactions will have no effect on the size of studding used.

End wind. A graphical analysis of this barn, with an end wind is shown in Figure 29. Table 21 shows a summary of moments, shears, and stresses determined in this analysis.

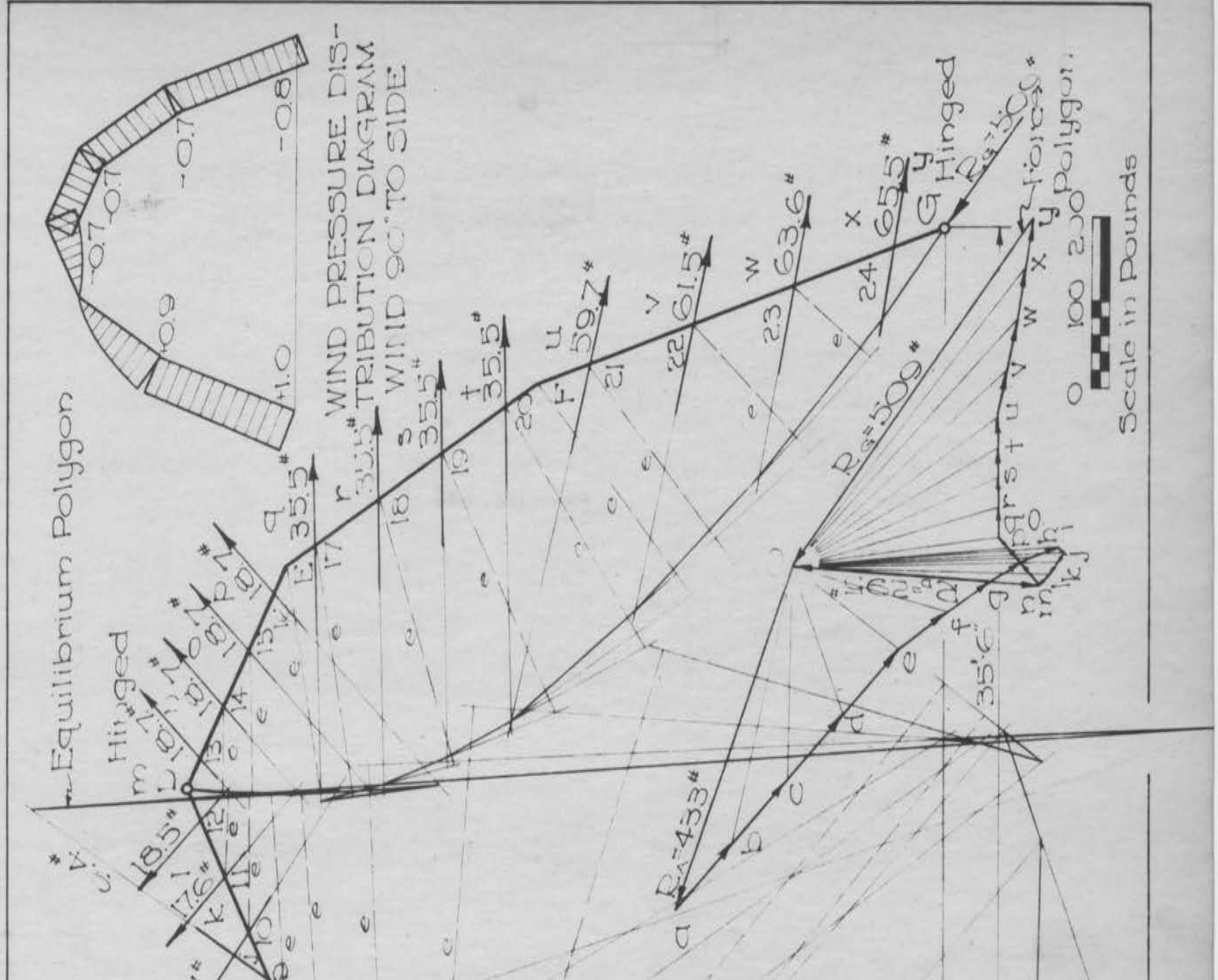


Fig.28-Stress Analysis-Combined Dead and Wind Loads
36' width - 3 rafter - Side wind

Table 20. Summary of Moments, Shears, and Stresses
Rafter Under Combined Dead and Wind Loads
36' Barn with 2"x6" x 14', 10',& 8' Rafter Members
Hinged at Plate and Ridge

Wind 90° to Side

Pt.	e in.	Thrust #	Moment in. #	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Direct Stress #	Direct Fiber Stress #/sq."
A	0	433	0	0	432	71.00	19	2.08
1	27.8	336	+ 9,340	1,090	331	54.35	53	5.80
2	96.0	251	+24,100	2,815	234	38.42	86	9.41
3	176.4	184	+32,450	3,790	139	22.84	119	13.02
4	242.0	159	+38,500	4,500	46	7.55	152	16.64
B	248.1	159	+39,460	4,610	5	0.82	158	17.30
5	207.2	190	+39,400	4,600	63	10.34	179	19.60
6	162.0	237	+38,400	4,486	125	20.54	200	21.90
7	121.0	282	+34,150	3,985	175	28.75	220	24.10
8	93.3	317	+29,600	3,455	204	33.50	240	26.26
C	84.1	317	+26,670	3,115	299	49.10	69	7.55
9	70.2	324	+22,050	2,575	303	49.70	111	12.15
10	52.8	318	+16,780	1,960	293	48.10	119	13.03
11	31.5	304	+ 9,710	1,133	276	45.35	127	13.91
12	11.1	295	+ 3,270	229	259	42.55	136	14.89
D	0	295	0	0	259	42.55	136	14.89
13	11.0	295	- 3,145	368	278	45.65	97	10.62
14	31.7	281	- 8,910	1,040	260	42.70	105	11.50
15	54.0	269	-14,500	1,693	243	39.90	112	12.27
16	78.0	257	-20,500	2,395	226	37.10	121	13.24
E	90.5	247	-22,360	2,610	208	34.15	128	14.02
17	96.7	247	-23,900	2,790	113	18.55	217	23.78
18	104.2	253	-26,400	3,082	85	13.96	233	25.52
19	106.4	264	-28,100	3,280	57	9.36	256	28.02
20	104.3	279	-29,100	3,400	28	4.60	277	30.32
F	97.8	279	-27,300	3,186	0	0	279	30.55
21	92.5	298	-27,560	3,216	74	12.15	288	31.52
22	64.8	343	-22,230	2,598	124	20.36	319	34.95
23	38.5	394	-15,170	1,770	177	29.05	352	38.55
24	12.9	448	- 5,780	675	230	37.76	384	42.05
G	0	509	0	0	288	47.30	417	45.67

Horizontal Reaction A = 411# Horizontal Reaction G = 420#
Vertical Reaction A = 137# Vertical Reaction G = 287#

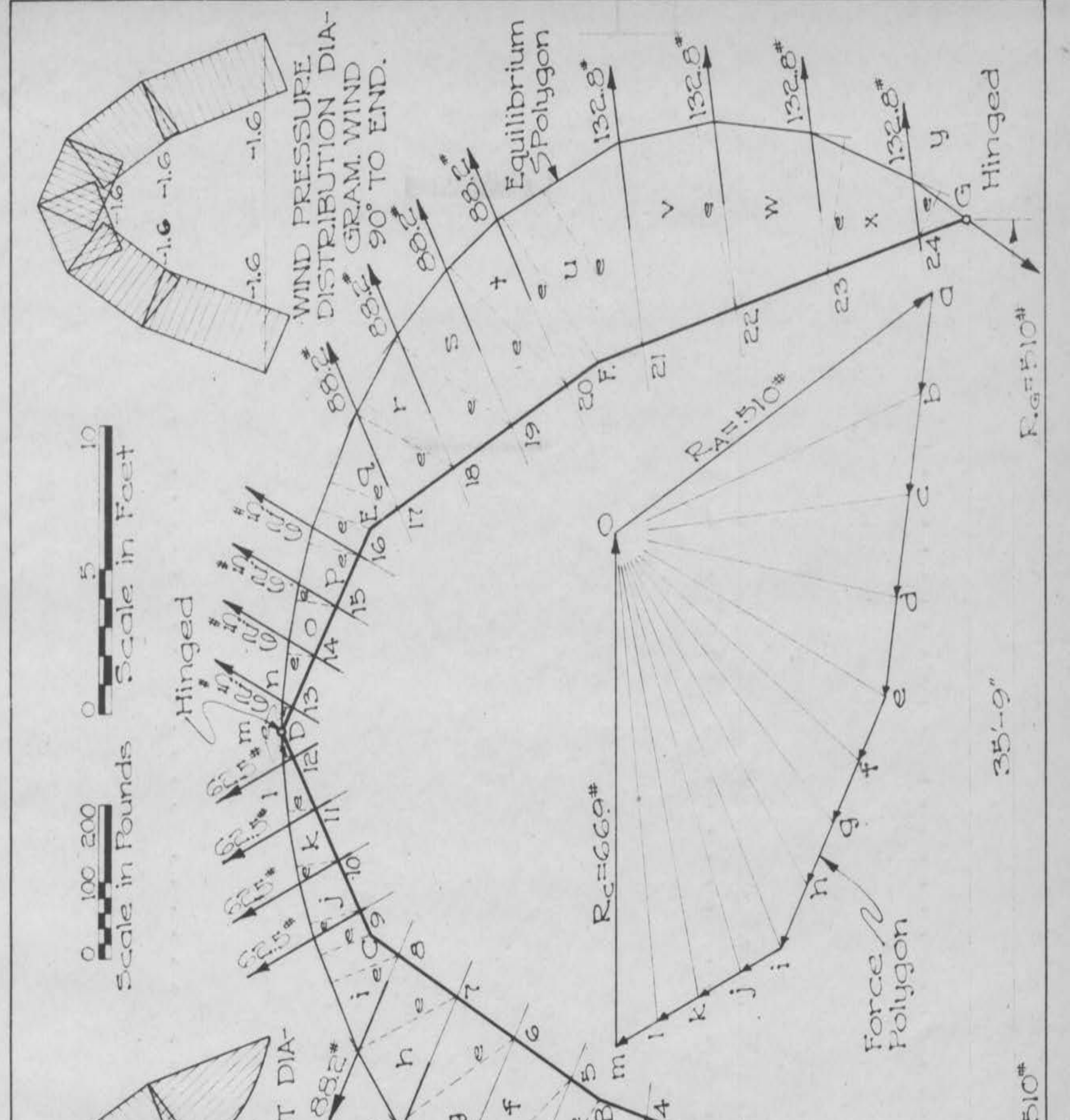


Fig.29-Stress Analysis-Combined Dead and Wind Loads.
36' width-3 rafter-End wind.

Table 21. Summary of Moments, Shears, and Stresses
Rafter Under Combined Dead & Wind Loads
36' Barn with 2"x6" x14', 10', & 8' Rafter Members
Hinged at Plate and Ridge

Wind 90° to End

Pt.	e in.	Thrust #	Moment in. #	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Stress #	Direct Fiber Stress #/sq."
A	0	510	0		442	72.60	274	29.98
1	20.7	436	- 9,025	1,054	312	51.23	307	33.60
2	57.0	383	-21,840	2,550	183	30.06	338	37.00
3	78.5	375	-29,450	3,440	53	8.71	372	40.70
4	76.5	411	-31,420	3,670	75	12.32	402	44.00
B	72.8	411	-29,900	3,492	32	5.26	408	44.65
5	69.6	432	-30,100	3,514	55	9.04	428	44.65
6	60.9	471	-28,660	3,347	142	23.32	448	49.00
7	46.7	522	-24,380	2,846	226	37.12	468	51.25
8	30.5	582	-17,760	2,073	312	51.23	488	53.40
C	22.6	582	-13,170	1,538	20	3.29	582	63.70
9	21.3	595	-12,680	1,480	82	13.46	588	64.40
10	17.7	614	-10,880	1,271	145	24.10	596	65.16
11	11.8	639	- 7,550	881	206	33.83	605	66.15
12	4.8	669	- 3,210	375	267	43.82	612	67.00
D	0	669	0	0	267	43.82	612	67.00

Horizontal Reaction A = 312#   Vertical Reaction A =-410#

Table 22. Summary of Moments, Shears, and Stresses
Rafter Under Combined Dead & Wind Loads
36' Barn with 2"x6" x 14', 12', & 8' Rafter Members
Hinged at Plate and Ridge

Wind 90° to End

A	0	533	0		480	78.85	235	25.72
1	22.6	441	- 9,960	1,163	350	57.50	269	29.44
2	65.5	374	-24,500	2,862	224	36.80	303	33.20
3	97.2	346	-33,630	3,930	92	15.10	334	36.56
4	101.8	368	-37,450	4,372	37	6.08	365	39.95
B	99.8	368	-36,740	4,286	53	8.70	360	39.40
5	96.0	393	-37,750	4,410	54	8.87	391	42.80
6	80.9	443	-35,850	4,180	154	25.30	414	45.32
7	60.3	510	-30,750	3,590	259	42.55	440	48.10
8	36.4	589	-21,440	2,505	361	59.30	466	51.00
C	25.8	589	-15,210	1,776	53	8.70	587	64.26
9	23.8	607	-14,460	1,688	111	18.22	596	65.18
10	19.0	629	-11,960	1,397	169	27.75	607	66.50
11	12.3	655	- 8,056	941	226	37.13	616	67.40
12	4.6	685	- 3,150	368	283	46.50	625	68.40
D	0	685	0	0	283	46.50	625	68.40

Horizontal Reaction A = 378#   Vertical Reaction A =-380#

The maximum bending moment in this barn roof is -31,420 in.lbs. at point 4 on the lower rafter member. This bending moment causes a stress of 3,670 #/sq.in. in the member. This fiber stress is too large for what is considered a safe allowable stress. By the use of a 2"x8" member, the stress would be reduced to 2,064 #/sq.in. The largest bending moment at the joint for this roof is -29,900 in. lbs. at joint B.

The largest horizontal shear in this rafter member is 72.6 #/sq.in.; the largest direct fiber stress is 67 #/sq.in. Neither of these stresses is an important factor in this design.

The reaction at the plates is 510#. The horizontal component of this reaction is 312#; the vertical component is -410#. The reaction at the ridge is 669#.

Combined dead and wind load stress analysis, 36' barn with 14',12', & 8' rafter members

Side wind. An illustration of a graphical analysis of this barn roof, with a side wind, is shown in Figure 30. A summary of the moments, shears, and stresses found in this analysis is presented in Table 23.

The maximum bending moment found is 45,250 in.lbs. at point 5. This moment creates a fiber stress of 5,281 #/sq. in., which is considerably over the allowable working stress of 3,000 #/sq.in. The use of a 2"x8" rafter would reduce

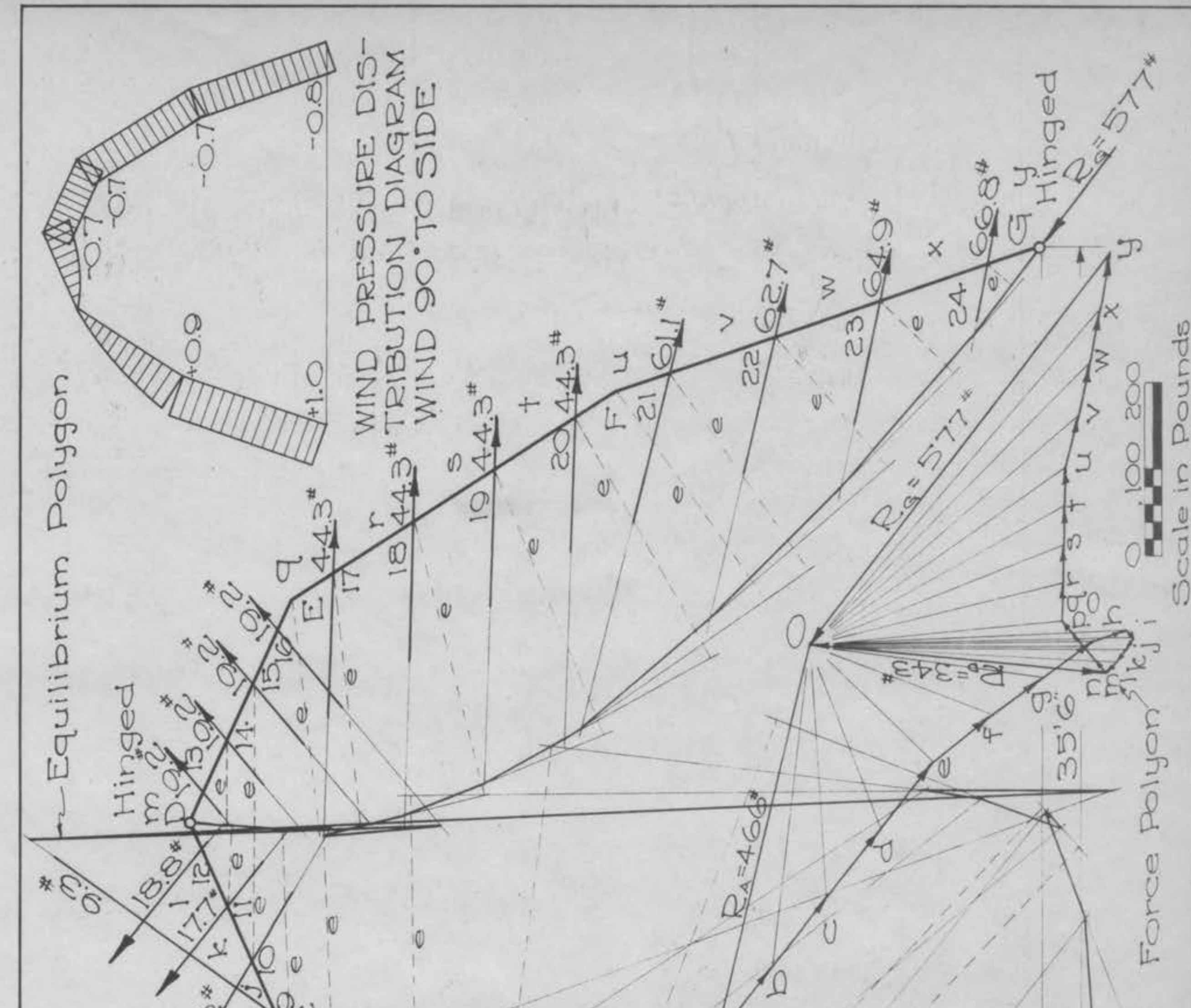


Fig.30-Stress Analysis-Combined Dead and Wind Loads
36' width - 3 rafter - Side wind

Table 23. Summary of Moments, Shears, and Stresses
Rafter Under Combined Dead and Wind Loads
36' Barn with 2"x6" x 14', 12' & 8' Rafter Members
Hinged at Plate and Ridge

Wind 90° to Side

Pt.	e in.	Thrust #	Moment in. #	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Direct Stress #	Direct Fiber Stress #/sq."
A	0	466	0	0	464	76.19	47	5.15
1	25.6	373	+ 9,550	1,115	365	59.95	79	8.65
2	85.4	291	+24,850	2,900	270	44.37	113	12.38
3	158.4	226	+35,800	4,177	172	28.25	146	15.99
4	219.3	196	+43,000	5,020	81	13.30	179	19.69
B	227.8	196	+44,630	5,210	34	5.58	194	21.25
5	202.0	224	+45,250	5,281	31	5.09	219	23.96
6	160.3	275	+44,100	5,150	121	19.87	247	27.05
7	121.2	325	+39,400	4,600	181	29.75	269	29.22
8	91.2	367	+33,450	3,905	215	35.30	297	32.55
C	80.9	367	+29,680	3,465	340	55.84	134	14.68
9	68.3	370	+25,250	2,947	343	56.35	143	15.66
10	49.2	365	+17,980	2,100	333	54.70	151	16.54
11	29.8	357	+10,640	1,242	317	52.05	161	17.63
12	9.4	343	+ 3,222	376	300	49.25	169	18.50
D	0	343	0	0	300	49.25	169	18.50
13	11.1	343	- 3,810	445	330	54.15	125	13.69
14	31.5	328	-10,330	1,208	306	50.25	122	13.37
15	52.7	317	-16,720	1,952	288	47.29	129	14.12
16	74.8	303	-22,650	2,645	283	46.50	138	15.11
E	87.6	296	-25,960	3,030	252	41.40	147	16.10
17	95.7	296	-28,350	3,310	131	21.50	262	28.68
18	104.0	302	-31,410	3,668	95	15.60	287	31.45
19	105.9	317	-33,600	3,824	59	9.69	311	34.05
20	101.8	336	-34,200	3,993	21	3.45	335	36.68
F	94.0	362	-34,040	3,975	15	2.46	361	39.50
21	88.5	362	-32,020	3,740	82	13.46	347	38.00
22	63.3	409	-25,900	3,025	147	24.15	383	41.90
23	37.9	463	-17,550	2,050	202	33.15	416	45.50
24	13.2	518	- 6,840	799	258	42.40	450	49.23
G	0	577	0	0	315	51.72	482	52.65

Horizontal Reaction A = 453# Horizontal Reaction G = 459#
Vertical Reaction A = 107# Vertical Reaction G = 350#

this stress to 2,970 #/sq.in. This rafter would safely resist the maximum bending moment due to this load. The largest bending moment at the rafter joints is 44,630 in. lbs. at joint B on the windward side.

The largest horizontal shear found in this analysis is 76.2. #/sq.in. The maximum direct stress is 52.7 #/sq.in. These stresses are not limiting factors in this design.

The largest reaction is 577# at plate G. The horizontal component of this reaction is 459#, and the vertical component is 350#. To resist these reactions it is necessary to secure some means of fastening these rafters to the plate other than toenailing. This can be done by using a rafter brace from the rafter to the studding.

<u>End wind</u>. A graphic analysis of this barn, with an end wind, is illustrated in Figure 31. A summary of the moments, shears, and stresses found in this analysis is presented in Table 22.

The maximum bending moment found in the analysis is -37,750 in. lbs. at point 5 on the middle rafter member. This moment produces a fiber stress of 4,410 #/sq.in. If the 2"x6" member was replaced by a 2"x8" member, the stress would be reduced to 2,478 #/sq.in. The maximum bending moment at the joints is -36,740 in. lbs. at joint B.

The largest horizontal shear in this rafter is 78.9 #/sq. in. The maximum direct stress is 68.4 #/sq.in. Neither of

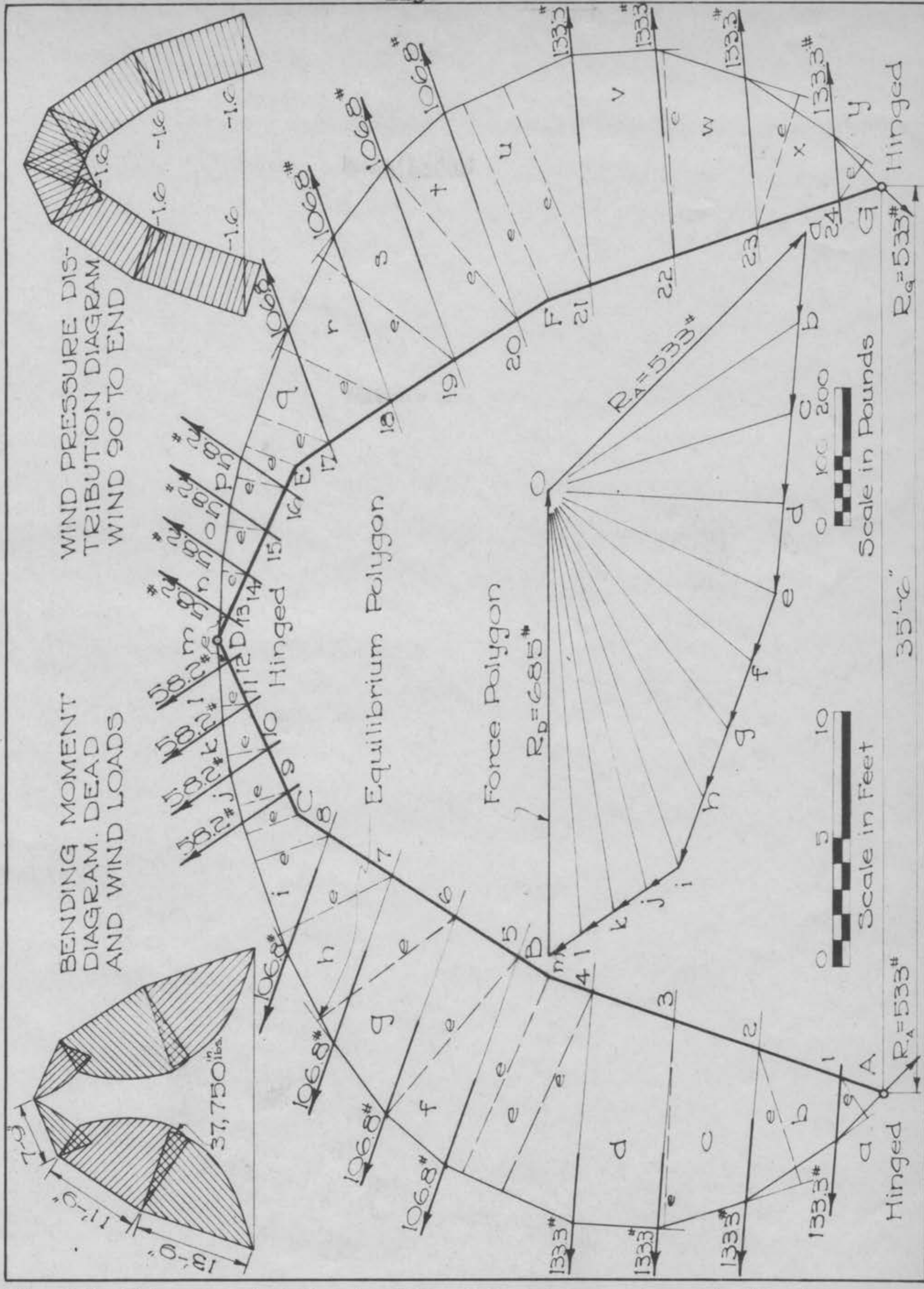


Fig.31-Stress Analysis-Combined Dead and Wind Loads
36' width-3 rafter- End wind

these stresses is large enough to consider in this design.

The reaction at the plates is 533 #. The horizontal component of this reaction is 378#. The vertical component is -380#.

Combined wind and dead load stress analysis, 40'barn with 14',12',& 8' rafters

The wind load data used in the stress analyses of the 40' barns are shown in Table 24. The dead load and wind load are combined into resultants, which are considered as acting at four equally divided sections on each rafter member.

Side wind. A graphical analysis of this barn roof, with a side wind, is illustrated in Figure 32. A summary of moments, shears, and stresses, determined in this analysis, is presented in Table 25.

The maximum bending moment found in this analysis is 45,150 in. lbs. at point 5 on the middle rafter member. The fiber stress created in a 2"x6" member by this moment is 5,270 #/sq.in. This stress is 75.7 per cent higher than the allowable stress of 3,000 #/sq.in. By the use of a 2"x8" member the stress is reduced to 2,960 #/sq.in. This member can be relied upon to carry the maximum moment created in this barn roof. The maximum bending moment at the joints is 44,350 in. lbs. at joint B.

Table 24. Wind Load Data for Three Rafter Gambrel Barn Roofs
40' Barn -70 M.P.H. Wind

Wind 90° to Side

Load No.	Coefficient	Pressure #/sq.'	14', 12', & 8' Rafter Members: Total P #	Dead Load #	Resultant #	14', 12', & 10' Rafter Members: Total P #	Dead Load #	Resultant #
1	0.987	12.55	86.7	35	105.2	86.7	35	104.3
2	0.963	12.55	84.7	35	103.1	84.7	35	102.4
3	0.937	12.55	82.3	35	101.2	82.3	35	100.0
4	0.913	12.55	80.3	35	99.0	80.3	35	98.3
5	0.870	12.55	65.5	30	87.2	65.5	30	85.6
6	0.745	12.55	56.1	30	78.2	56.1	30	76.6
7	0.555	12.55	41.8	30	64.8	41.8	30	63.5
8	0.230	12.55	17.3	30	42.9	17.3	30	42.0
9	-0.265	12.55	13.3	20	8.8	16.6	25	12.6
10	-0.596	12.55	29.9	20	13.2	37.4	25	18.9
11	-0.675	12.55	33.9	20	16.7	42.4	25	23.0
12	-0.695	12.55	34.9	20	17.6	43.6	25	24.1
13	-0.70	12.55	35.2	20	17.8	43.9	25	24.3
14	-0.70	12.55	35.2	20	17.8	43.9	25	24.3
15	-0.70	12.55	35.2	20	17.8	43.9	25	24.3
16	-0.70	12.55	35.2	20	17.8	43.9	25	24.3
17	-0.70	12.55	52.8	30	41.6	52.8	30	44.5
18	-0.70	12.55	52.8	30	41.6	52.8	30	44.5
19	-0.70	12.55	52.8	30	41.6	52.8	30	44.5
20	-0.70	12.55	52.8	30	41.6	52.8	30	44.5
21	-0.713	12.55	62.7	35	59.4	62.7	35	60.4
22	-0.737	12.55	64.7	35	60.8	64.7	35	62.0
23	-0.763	12.55	67.1	35	62.9	67.1	35	64.5
24	-0.787	12.55	69.2	35	64.5	69.2	35	66.1

Wind 90° to End

1	-1.6	12.55	141.0	35	131:7	141.0	35	133.2
2	-1.6	12.55	141.0	35	131.7	141.0	35	133.2
3	-1.6	12.55	141.0	35	131.7	141.0	35	133.2
4	-1.6	12.55	141.0	35	131.7	141.0	35	133.2
5	-1.6	12.55	120.0	30	102.2	120.0	30	105.1
6	-1.6	12.55	120.0	30	102.2	120.0	30	105.1
7	-1.6	12.55	120.0	30	102.2	120.0	30	105.1
8	-1.6	12.55	120.0	30	102.2	120.0	30	105.1
9	-1.6	12.55	80.3	20	62.1	100.5	25	70.8
10	-1.6	12.55	80.3	20	62.1	100.5	25	70.8
11	-1.6	12.55	80.3	20	62.1	100.5	25	70.8
12	-1.6	12.55	80.3	20	62.1	100.5	25	70.8

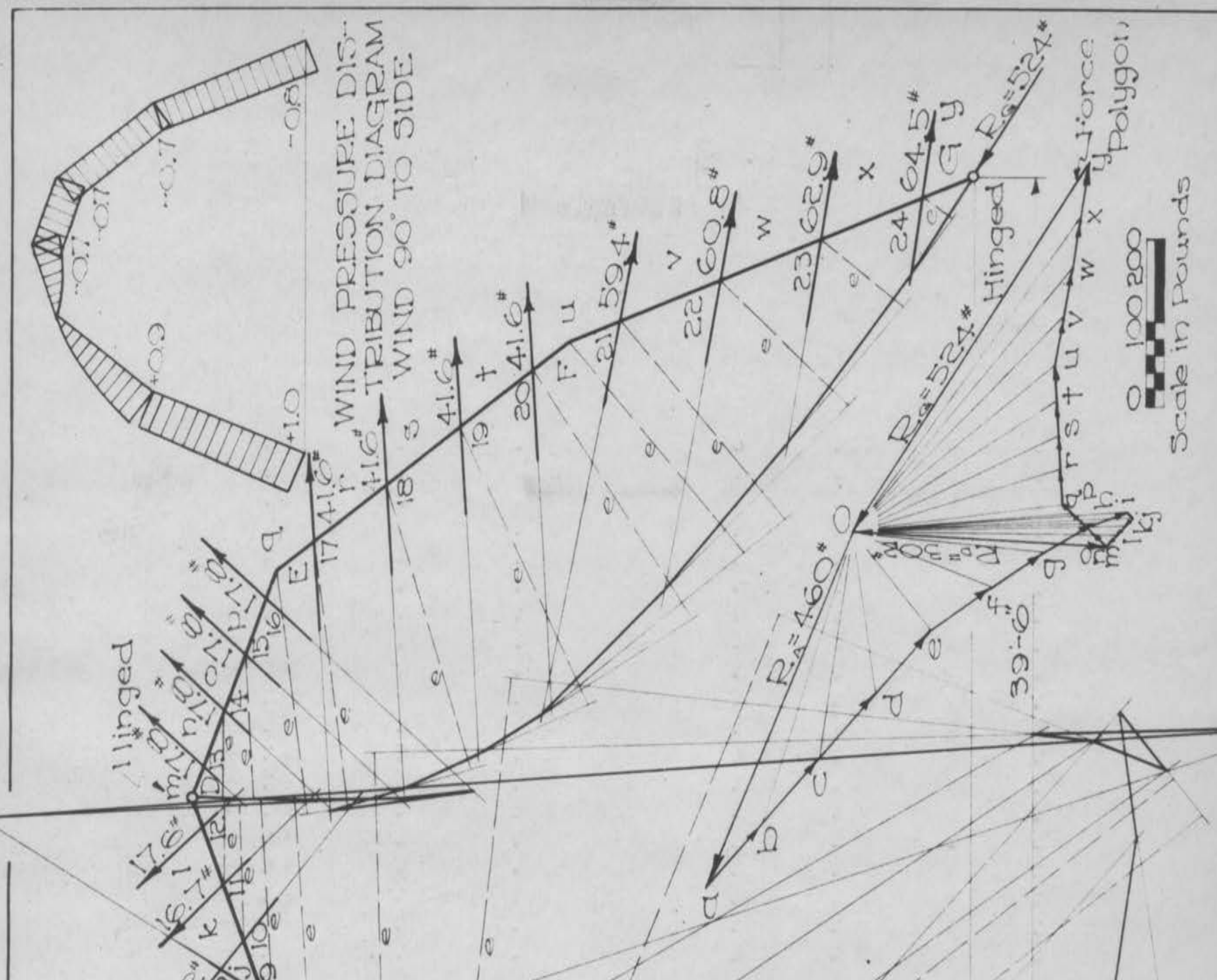


Fig. 32-Stress Analysis-Combined Dead and Wind Loads
40' width - 3 rafter - Side wind

Table 25. Summary of Moments, Shears, and Stresses
Rafter Under Combined Dead and Wind Loads
40' Barn with 2"x6" x 14', 12' and 8' Rafter Members
Hinged at Plate and Ridge

Wind 90° to Side

Pt.	e in.	Thrust #	Moment in. #	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Direct Stress #	Direct Fiber Stress #/sq."
A	0	460	0	0	460	75.50	5	0.55
1	26.7	362	+ 9,300	1,086	361	59.20	33	3.61
2	90.7	271	+24,600	2,873	265	43.50	66	7.22
3	183.6	195	+35,800	4,180	169	27.67	99	10.83
4	283.3	151	+42,760	4,990	73	11.99	132	14.44
B	293.6	151	+44,350	5,175	34	5.58	148	16.20
5	252.0	179	+45,150	5,270	50	8.21	171	18.72
6	188.5	231	+43,550	5,083	123	20.20	194	21.25
7	137.0	287	+39,300	4,590	183	30.06	218	23.85
8	98.4	326	+32,070	3,740	220	36.15	241	26.40
C	86.6	326	+28,220	3,295	313	51.40	92	10.06
9	73.5	335	+24,620	2,875	319	52.40	97	10.61
10	54.0	335	+18,100	2,114	307	50.40	104	11.38
11	32.2	315	+10,150	1,085	293	48.10	113	12.37
12	10.8	302	+ 3,260	381	277	45.50	120	13.13
D	0	302	0	0	277	45.50	120	13.13
13	10.9	302	- 3,290	384	288	47.30	87	9.52
14	33.2	289	- 9,600	1,121	371	60.95	94	10.28
15	54.8	289	-15,840	1,850	253	41.55	102	11.16
16	76.7	275	-21,080	2,460	237	38.93	109	11.93
E	93.5	251	-23,460	2,740	222	36.45	116	12.70
17	101.8	251	-25,550	2,983	128	21.00	214	23.42
18	112.6	257	-28,950	3,380	93	15.27	238	26.05
19	115.8	270	-31,260	3,650	61	10.02	261	28.55
20	112.5	284	-31,960	3,730	26	4.27	283	30.97
F	102.3	311	-31,850	3,720	10	1.64	310	33.92
21	97.5	311	-30,380	3,545	95	15.60	296	32.40
22	67.8	358	-24,260	2,830	144	23.65	326	35.70
23	40.2	410	-16,480	1,923	194	31.85	360	39.40
24	13.7	465	- 6,365	744	248	40.70	392	42.90
G	0	524	0	0	306	50.28	424	46.40

Horizontal Reaction A = 423# Horizontal Reaction G = 442#
Vertical Reaction A = 172# Vertical Reaction G = 282#

The maximum horizontal shear found in this analysis is 75.5 #/sq.in. The maximum direct stress is 46.4 #/sq.in. These stresses are not important factors in this design.

The largest reaction at the plates is 524# at plate G on the leeward side. The horizontal component of this reaction is 442#. The vertical component is 242#. The reaction at the ridge is 302#.

End wind. A graphic analysis of this barn roof, with an end wind, is shown in Figure 33. A summary of the moments, shears, and stresses, determined in this analysis, is presented in Table 26.

The maximum bending moment found is -32,580 in. lbs. This moment creates a fiber stress of 3,805 #/sq.in, in a 2"x6" member. This member is not large enough to safely resist this load. By using a 2"x8" member the stress is reduced to 2,140 #/sq.in. The 2"x8" member can be relied upon to carry the load for this barn roof. The maximum moment at the rafter joints is -31,050 in. lbs. at joint B.

The largest horizontal sh ar in this roof is 72.7 #/sq.in. The largest direct fiber stress is 73.5 #/sq.in. These stresses are of no concern in this design.

The reaction at the plates is 541#. The horizontal component of this reaction is 293#. The vertical component is -455#. The reaction at the ridge is 713#.

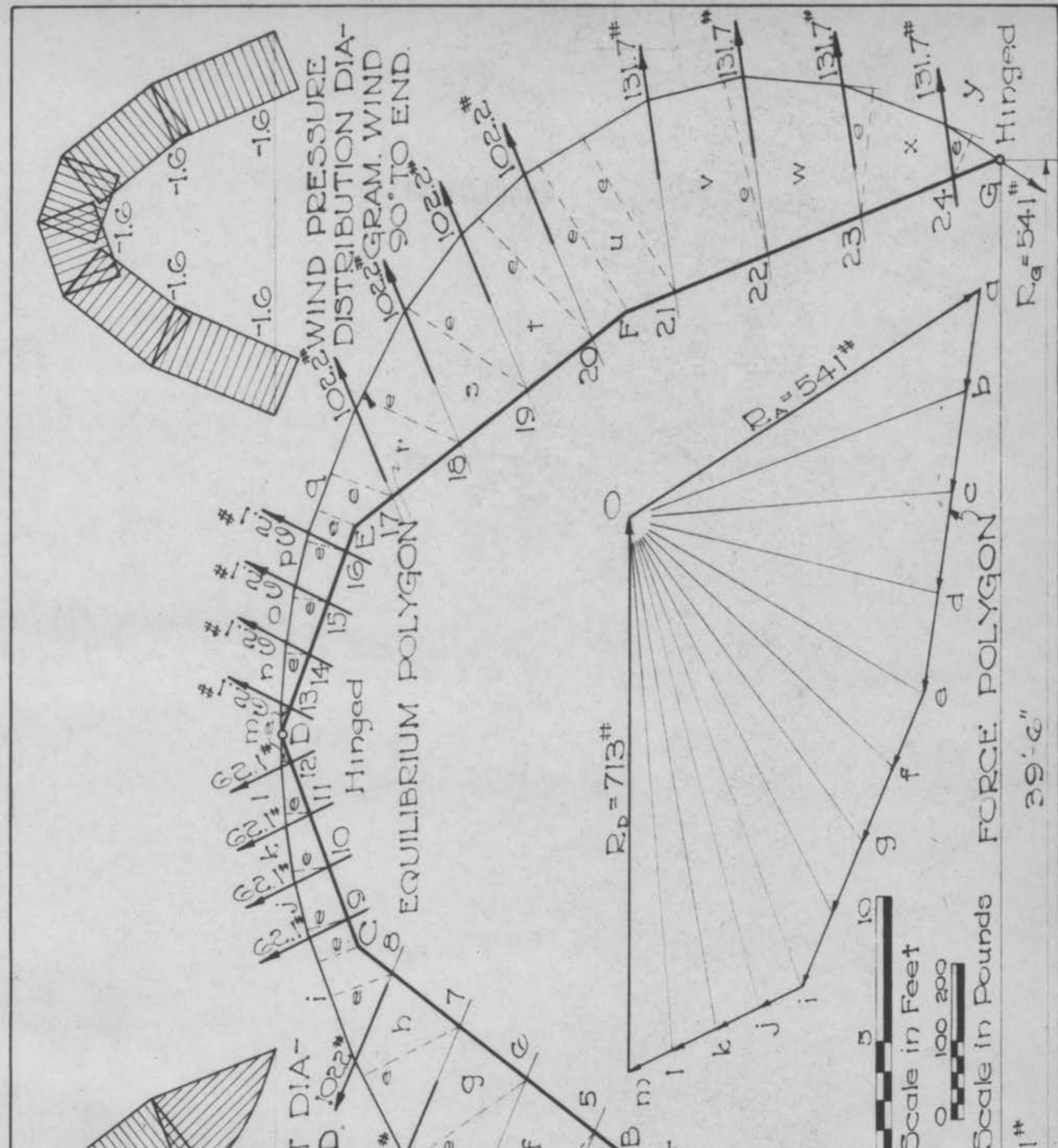


Fig.33--Stress Analysis--Combined Dead and Wind Loads.
40'width-3 rafter-End wind.

Table 26. Summary of Moments, Shears, and Stresses
Rafter Under Combined Dead and Wind Loads
40' Barn with 2"x6" x 14', 12' & 8' Rafter Members
Hinged at Plate and Ridge

Wind 90° to End

Pt.	e in.	Thrust #	Moment in. #	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Direct Stress #	Direct Fiber Stress #/sq."
A	0	541	0	0	443	72.74	311	34.04
1	19.8	465	- 9,210	1,076	315	51.72	342	37.45
2	53.8	418	-22,280	2,600	187	30.70	374	40.90
3	73.6	410	-30,190	3,526	59	9.69	406	44.40
4	73.5	443	-32,580	3,805	68	11.17	437	47.85
B	70.1	443	-31,050	3,625	56	9.19	440	48.13
5	68.3	468	-31,950	3,730	42	6.80	466	51.00
6	59.3	513	-30,200	3,525	142	23.30	494	54.10
7	44.6	573	-25,550	2,983	240	39.40	521	57.00
8	26.2	642	-16,830	1,966	337	55.34	547	59.85
C	17.2	644	-11,080	1,292	4	0.66	642	70.25
9	17.0	652	-11,080	1,292	57	9.36	651	71.24
10	14.8	667	- 9,880	1,154	118	19.38	658	72.00
11	9.9	687	- 6,800	794	180	29.55	665	72.80
12	3.8	713	- 2,710	316	241	39.60	671	73.50
D	0	713	0	0	241	39.60	671	73.50

Horizontal Reaction A = 293# Vertical Reaction A =-455#

Table 27. Summary of Moments, Shears, and Stresses
Rafter Under Combined Dead and Wind Loads
40' Barn with 2"x6" x 14', 12' and 10' Rafter Members
Hinged at Plate and Ridge

Wind 90° to End

A	0	553	0	0	498	81.77	243	26.60
1	22.6	459	-10,380	1,212	387	63.53	275	30.10
2	66.2	388	-25,680	3,000	240	39.40	307	33.60
3	100.4	356	-35,750	4,170	111	18.22	339	37.10
4	108.5	374	-40,600	4,740	17	2.79	372	40.70
B	107.0	374	-40,040	4,673	66	10.84	365	39.96
5	103.7	396	-41,100	4,800	33	5.42	394	43.12
6	89.1	445	-39,650	4,630	136	22.32	424	46.40
7	68.3	512	-35,000	4,087	236	38.75	453	49.60
8	44.8	588	-26,370	3,077	337	55.35	483	52.85
C	33.6	588	-19,780	2,307	48	7.87	586	64.10
9	31.0	613	-19,000	2,218	114	18.70	601	65.75
10	25.8	643	-16,580	1,935	183	30.05	616	67.40
11	16.4	681	-11,180	1,307	251	41.20	631	69.00
12	6.0	722	- 4,330	506	321	52.70	646	70.70
D	0	722	0	0	321	52.70	646	70.70

Horizontal Reaction A = 381# Vertical Reaction A =-396#

Combined wind and dead load stress analysis, 40' barn with 14', 12', & 10' rafters

Side wind. A graphic analysis of this barn roof, with a side wind, is shown in Figure 34. A summary of the moments, shears, and stresses determined in this analysis is presented in Table 28.

The maximum bending moment found is 47,250 in.lbs. at point 5 on the middle rafter member. This moment results in a fiber stress of 5,520 #/sq.in. when a 2"x6" rafter member is used. By using a 2"x8" member, the stress would be reduced to 3,100 #/sq.in. This is slightly greater than the allowable of 3,000 #/sq.in., but the difference is not large enough to warrant the use of a 2"x10" member. The maximum bending moment at the joints is 46,100 in. lbs. at joint B.

The maximum horizontal shear is 77.4 #/sq.in., and the maximum direct stress is 48.4#/sq.in. These stresses are not large enough to consider in this design.

The largest reaction at the plates is 548# at plate G. The horizontal component of this reaction is 454#; the vertical component is 305#. The reaction at the ridge is 130#.

End wind. A graphical stress analysis of this barn roof, with an end wind, is illustrated in Figure 35. A summary of moments, shears, and stresses, determined in this analysis,

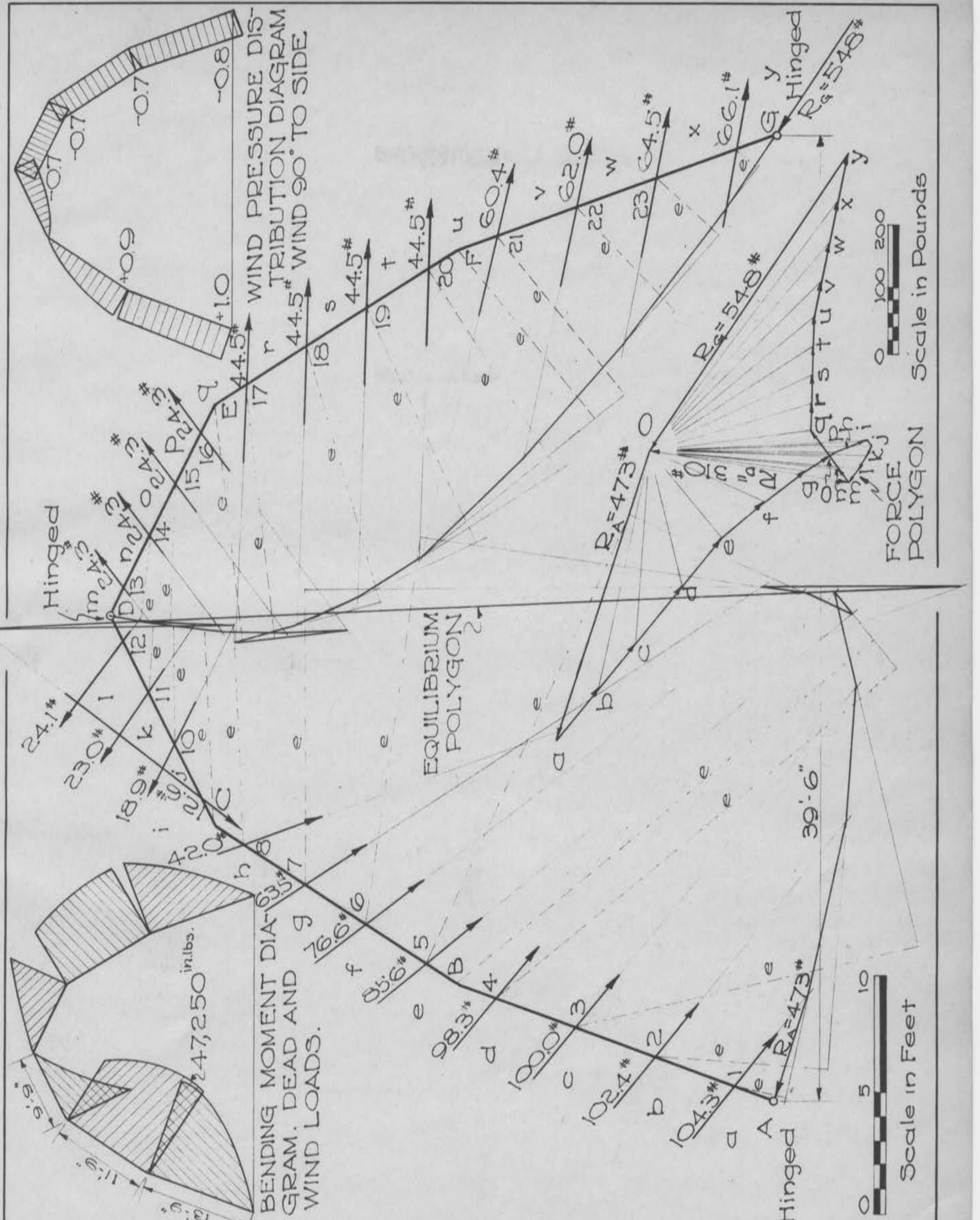


Fig.34-Stress Analysis-Combined Dead and Wind Loads.
40' width-3 rafter-Side wind.

Table 28. Summary of Moments, Shears, and Stresses
Rafter Under Combined Dead and Wind Loads
40' Barn with 2"x6" x 14', 12' & 10' Rafter Members
Hinged at Plate and Ridge

Wind 90°to side

Pt.	e in.	Thrust #	Moment in.#	Bending Fiber Stress #/sq."	Vertical Shear #	Horizontal Shear #/sq."	Direct Stress #	Direct Fiber Stress #/sq."
A	0	473	0	0	471	77.40	18	1.97
1	25.4	376	+9,550	1,115	342	56.16	53	5.80
2	84.5	299	+25,260	2,950	287	47.12	81	8.87
3	172.0	216	+37,160	4,340	181	29.74	118	12.92
4	254.0	174	+44,200	5,160	87	14.29	152	16.63
B	264.7	174	+46,100	5,385	50	8.21	167	18.29
5	242.5	195	+47,250	5,520	30	4.93	193	21.14
6	191.3	241	+46,100	5,390	103	16.92	217	23.76
7	145.3	292	+42,400	4,950	161	26.45	243	26.66
8	111.0	333	+36,950	4,314	196	32.20	269	29.45
C	100.5	333	+33,450	3,906	304	49.93	135	14.78
9	72.8	343	+24,950	2,915	312	51.23	140	15.32
10	58.0	333	+19,350	2,260	310	50.90	145	15.88
11	33.2	323	+10,720	1,252	276	45.35	169	15.43
12	11.0	310	+ 3,410	398	253	41.44	175	19.15
D	0	310	0	0	253	41.55	181	19.82
13	14.6	310	-4,530	529	293	48.10	95	10.42
14	41.3	295	-12,190	1,423	274	45.00	102	11.17
15	70.2	278	-19,510	2,280	253	41.60	115	12.60
16	75.6	263	-19,880	2,320	228	37.45	130	14.23
E	116.0	250	-29,000	3,385	211	34.65	135	14.78
17	123.8	250	-30,960	3,615	158	25.95	226	24.75
18	129.4	261	-33,780	3,945	73	11.99	251	27.47
19	126.0	277	-34,920	4,075	36	5.91	276	30.22
20	116.7	300	-35,030	4,095	0	0	300	32.84
F	105.5	328	-34,640	4,050	36	5.91	323	35.38
21	99.0	328	-32,500	3,795	107	17.57	309	33.83
22	69.5	377	-26,200	3,060	157	25.79	343	37.57
23	40.7	430	-17,550	2,050	208	34.18	376	41.10
24	14.2	488	- 6,930	809	265	43.50	409	44.85
G	0	548	0	0	322	52.90	442	48.40

Horizontal Reaction A = 450# Horizontal Reaction G = 454#
Vertical Reaction A = 140# Vertical Reaction G = 305#

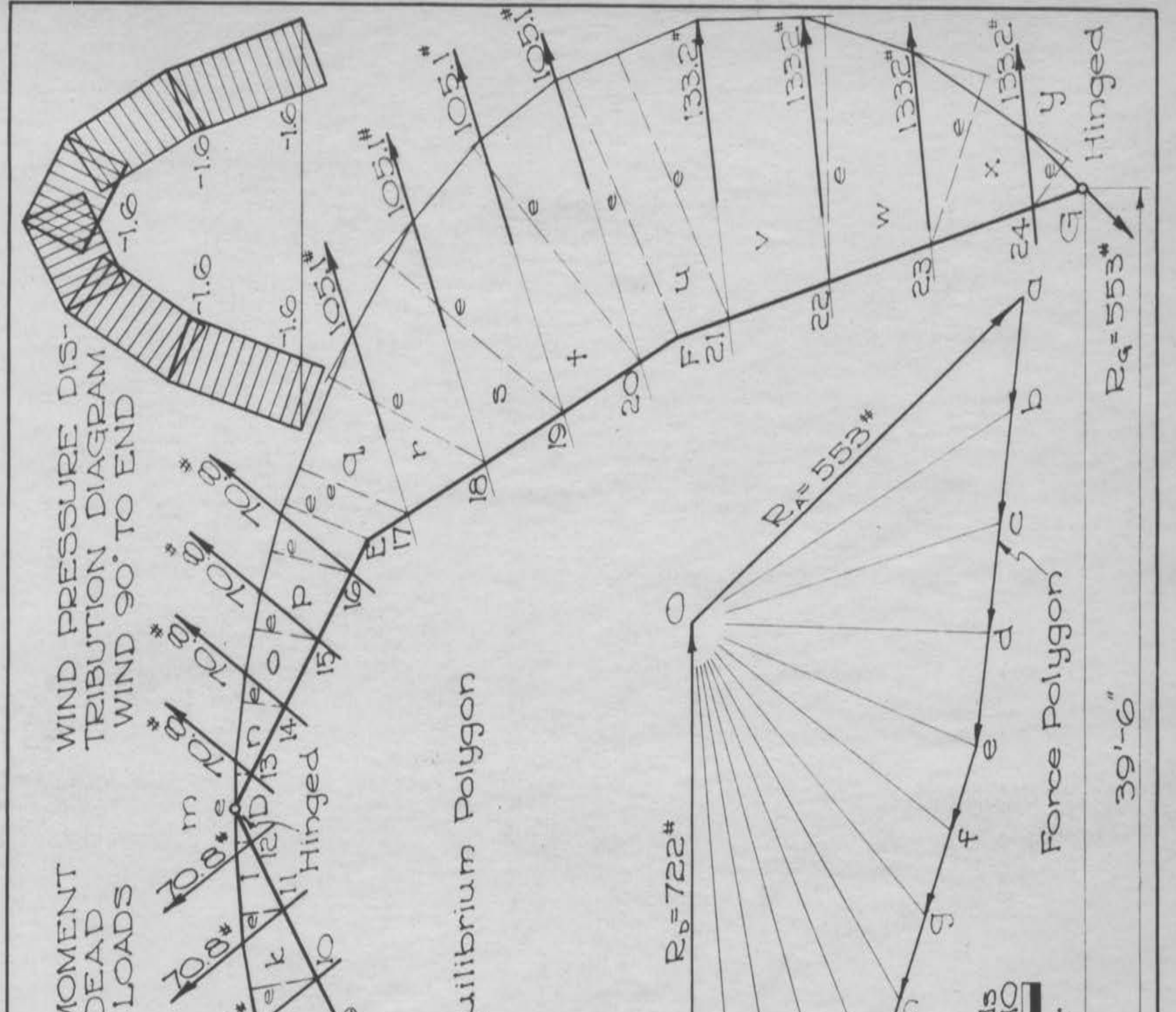


Fig.35-Stress Analysis-Combined Dead and Wind Loads
40' width - 3 rafter - End wind

is presented in Table 27.

The maximum bending moment found is -41,100 in. lbs. at point 5 on the middle rafter member. This moment creates a fiber stress of 4,800 #/sq.in. in a 2"x6" member. By the use of a 2"x8" member the stress is reduced to 2,695 #/sq. in., which is a safe stress for this loading. The maximum moment at the rafter joints is -40,040 in. lbs. at joint B.

The maximum horizontal shear in this roof is 81.8 #/sq.in. These stresses are too low to be alimiting factor in this design.

The reaction at the plates is 553#. The horizontal component of this reaction is 381#; the vertical component is -396#. The reaction at the ridge is 722#.

Plans and Specifications for Rafters

Plans

After having analyzed roofs for the 32', 34', 36', and 40' barns, detail plans were drawn for the rafters of each roof. The plans for the rafters are shown in Figures 36,37, and 38.

The rafters for the two rafter gambrel roofs are all similar in design. This is due to the fact that the maximum bending moments for these roofs occur in approximately the same place. The range of the large moments where bracing is needed is approximately the same. By the use of both the brace and the plate it is possible to secure a joint that is sufficiently strong to carry the necessary moment. These designs are similar to the rafter designed by Pickard (22) which failed only after a load equivalent to that produced by a 245 M.P.H. wind was applied. However, the amount of material used in these rafters is considerably smaller and the method of construction has been simplified.

The rafters for the three rafter gambrel roofs have no significant difference. The maximum moments on these roofs are approximately in the same places. These designs are similar to the rafter designed by Rice (23), which failed when a load equivalent to that produced by a 120 M.P.H. wind was applied. However, analyses in this investigation showed

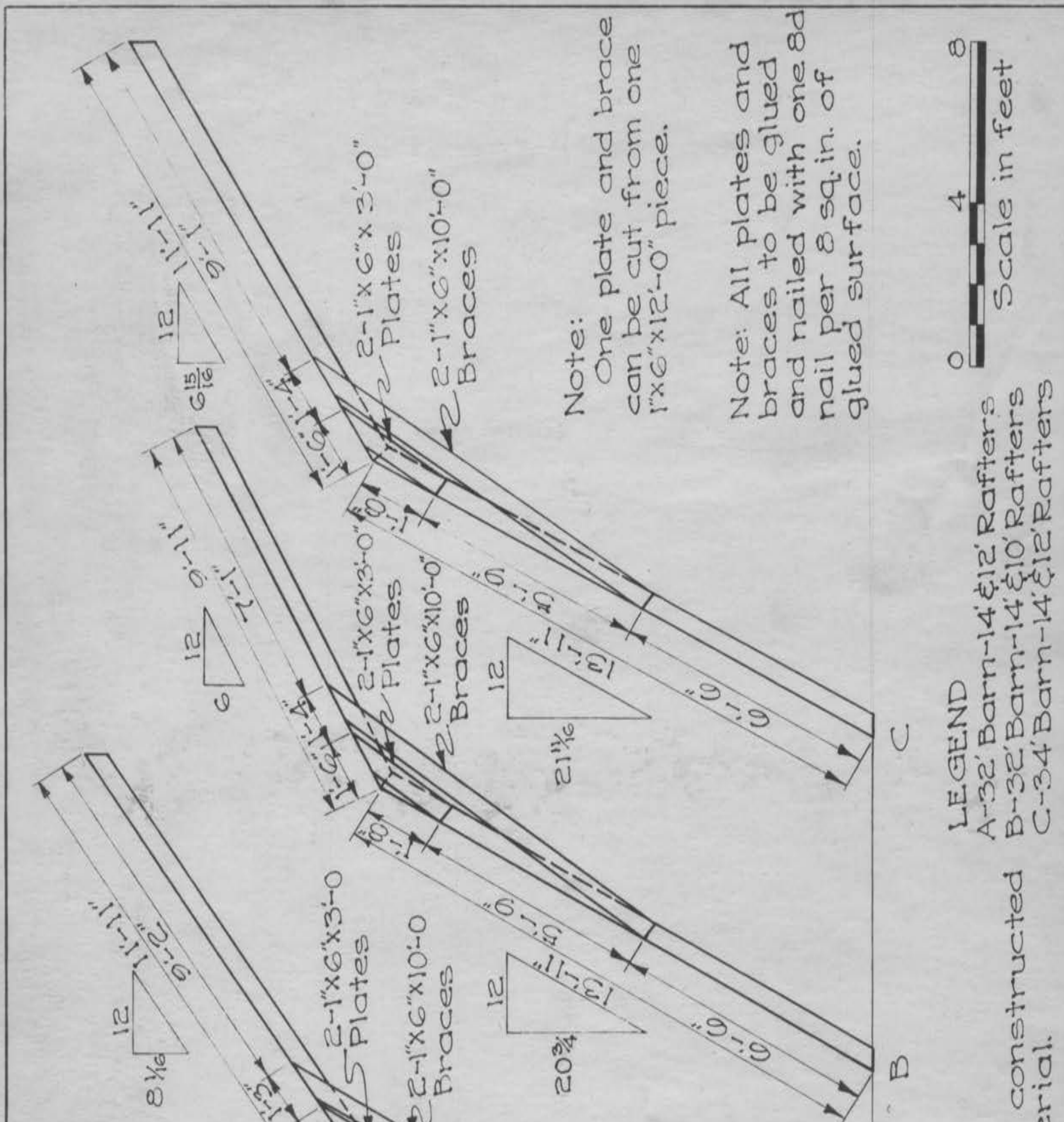


Fig. 36. Detail Plan of Rafters- 32' & 34' widths

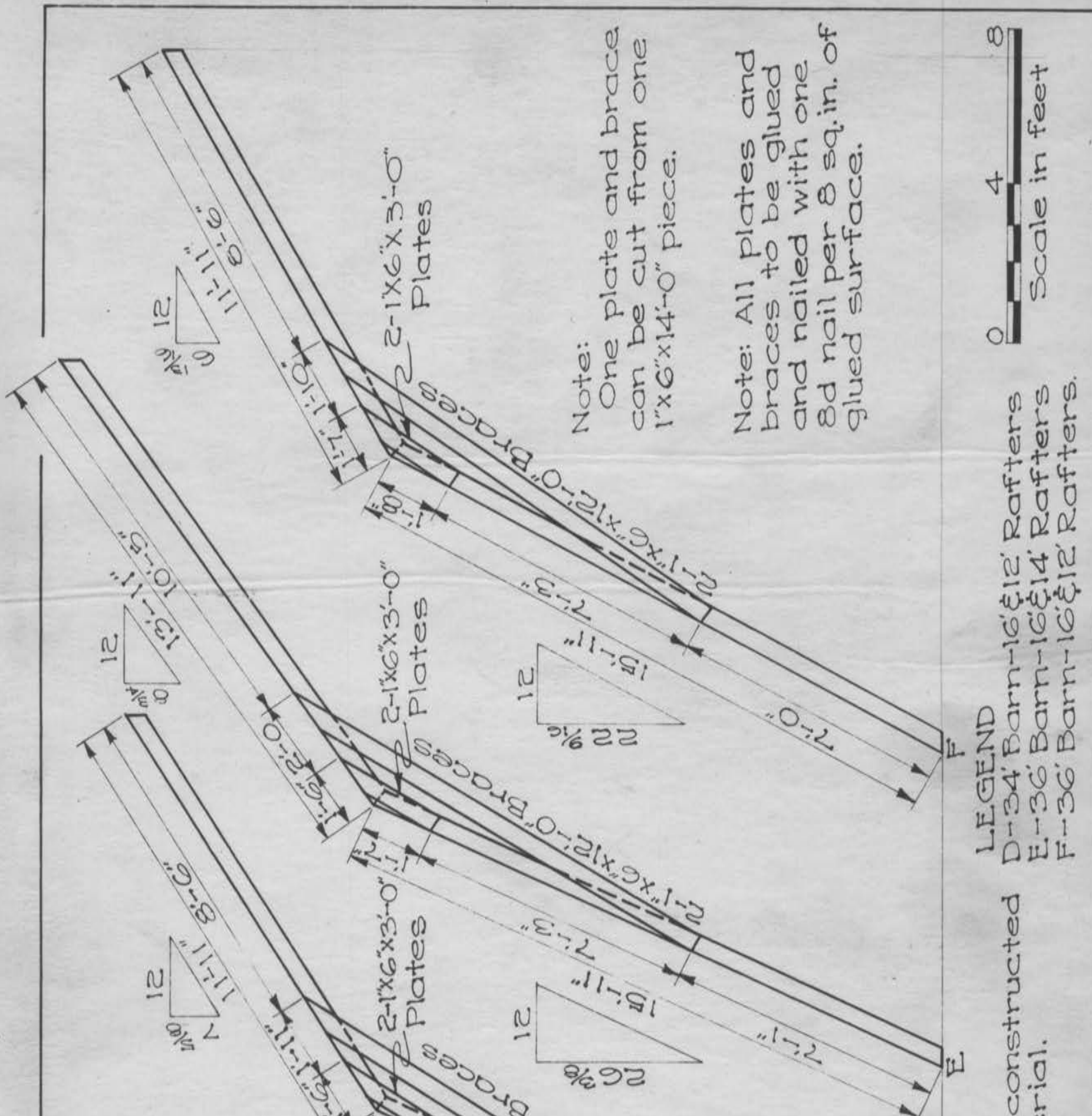


Fig. 37. Detail Plan of Rafters- 34' & 36' widths.

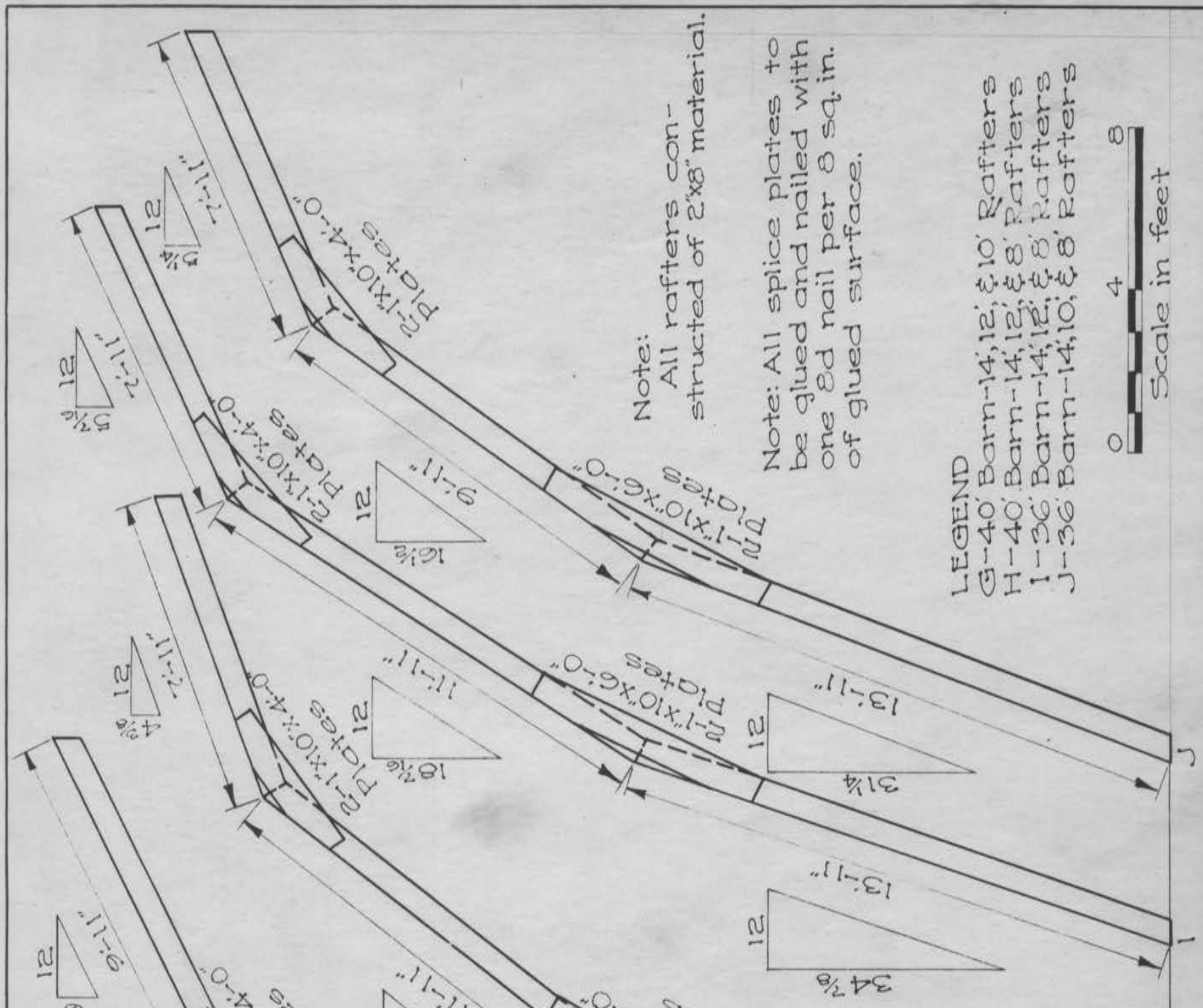


Fig. 38. Detail Plan of Rafters-36' & 40' widths.

that a 2"x6" member was not large enough to carry a load produced by a 70 M.P.H. without over stressing the members. As a result, 2"x8" members were selected for the designs. The use of this size material reduced the fiber stresses below the allowable of 3,000 #/sq.in. The size of the splice plates in these designs is considerably larger. Rice found that by the use of glued splice plates it was possible to secure a joint that approached the strength of a continuous member. Therefore, the splice plates used in these designs will safely carry the required loads.

Specifications

The specification for rafters for gambrel roofs may be listed as follows:

Material

1. Lumber to be of a good structural timber commonly used for framing.

2. The grade should not be lower than No. 2 common.

3. Pieces with defects in the outer fibers should not be used.

Gluing

1. Glue to be self-bonding, water resistant, cold water casein mixed according to manufacturer's specifications.

2. Glue to be applied to one side of the two pieces of material forming the joint.

Nailing

1. Nailing shall be done immediately after the application of the glue.

2. At least 1 - 8d nail or its equivalent must be provided for each 8 square inches of glued surface.

Discussion of specifications

A brief discussion will explain why certain specifications were made.

The grade No. 2 common was selected as it is believed that this grade of material will withstand all loads to which it may be subjected. Defects in the outer fibers may cause failure as the outer fibers of a beam have the greatest stress for this type of loading.

Casein glue is recommended as it is water resistant and may be obtained at a reasonable cost. It may be mixed with water and it dries quickly. Directions for the use of this glue should be closely followed. When first mixed, this glue is too stiff to use. After standing for approximately twenty minutes, it becomes thin and readily usuable.

In order to secure a good glued joint it is necessary to press the members together with a relatively light pressure. This can be accomplished by nailing the members together with 1 - 8d nail or its equivalent for each 8 square inches of glued surface.

Construction details

Construction details for framing a barn using a gambrel roof are shown in Figure 39. At the plate joint the rafter is toenailed to the plate and fastened to the studding by the means of a tie. At the ridge two 1"x6" pieces are used for collar beams. The use of glue at all the joints will greatly increase the strength and stiffness of the roof.

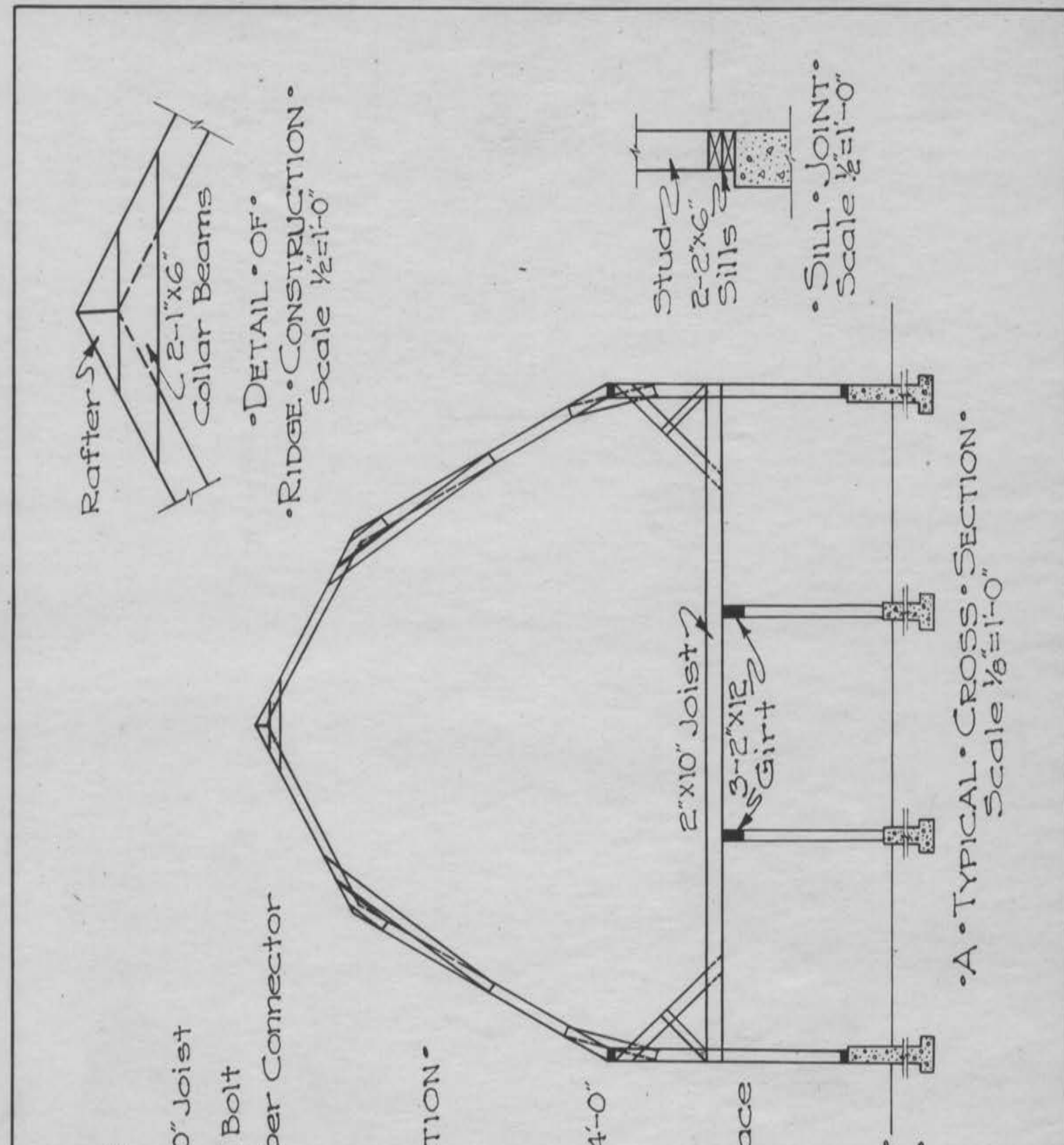


Fig. 39. Details of Construction.

SUMMARY

1. This study was justified by: (a) value of farm buildings; (b) data on wind damage; (c) importance of wood as a building material; (d) inadequacy of present gambrel barn roof designs.
2. Previous investigations of barn roof designs were reviewed.
3. The service, structural, economic, and aesthetic requirements of the barn roof were discussed.
4. The determination of barn sizes was discussed.
5. The methods of fastening wood joints were reviewed.
6. Wind pressures on gambrel roofs were investigated and the wind pressure distribution diagrams to be used in this study were selected.
7. Stable shapes were determined for roofs for the 32', 34', 36', and 40' widths of barns.
8. The rafters were assumed to be three hinged arches in this investigation.
9. The method used in making analyses of the barn roofs was discussed.
10. The allowable working stresses for wood were reviewed.
11. Stable roof shapes and Wooley's recommended pitches of 6/7 and 7/24 were analysed and compared structurally.
12. Dead load stress analyses of barn roofs were made for the

following:

(a) A 34' barn with 14' and 12' rafter members

(b) A 36' barn with 16' and 12' rafter members

(c) A 36' barn with 14', 10', and 8' rafter members

(d) A 40' barn with 14', 12', and 8' rafter members

13. Stress analyses of roofs under combined dead and wind loads were made for the following:

(a) A 32' barn with 14' and 10' rafter members

(b) A 32' barn with 14' and 12' rafter members

(c) A 34' barn with 14' and 12' rafter members

(d) A 34' barn with 16' and 12' rafter members

(e) A 36' barn with 16' and 12' rafter members

(f) A 36' barn with 16' and 14' rafter members

(g) A 36' barn with 14', 10', and 8' rafter members

(h) A 36' barn with 14', 12', and 8' rafter members

(i) A 40' barn with 14', 12', and 8' rafter members

(j) A 40' barn with 14', 12', and 10' rafter members

14. The bending moments, shears, and stresses for each of the barns listed in 13 were discussed.

15. Plans of rafters were drawn for each barn analyzed.

CONCLUSIONS

1. The tendency for the ridge of the gambrel roof to sag may be attributed to the following main causes:

   (a) Improper roof shape

   (b) Inadequate and improper bracing

2. Bending moments at the joints of rafter members, using Wooley's recommended pitches of 6/7 and 7/24, may cause deflection of the joints which result in sagging along the ridge.

3. There is no bending moment at the rafter joints when stable shapes are used for the roof under dead load.

4. The bending moments are larger in the middle of the rafter member with Wooley's recommended pitches than rafters with stable shapes.

5. The general standard roof pitches do not utilize standard length material.

6. Stable roof shapes can be selected that use standard length material for any width of barn.

7. Moments, shears, and stresses created by dead loads on stable shape roofs are not important factors in design of roofs.

8. The bending moments caused by a 70 M.P.H. wind on the barn roofs analyzed are as follows:

(a) In the 32' barn with 14' and 10' members, the maximum bending moment produced is 16,620 in. lbs. The maximum fiber stress developed is 1,942 #/sq.in.

(b) In the 32' barn with 14' and 12' members the maximum bending moment produced is 21,350 in.lbs. The maximum fiber stress is 2,495 #/sq.in.

(c) The maximum bending moment produced in the 34' barn with 14' and 12' members is 18,900 in. lbs. The fiber stress created by this moment is 2,210 #/sq. in.

(d) The maximum bending moment produced in the 34' barn with 16' and 12' members is 24,800 in. lbs. The fiber stress created by this moment is 2,898 #/sq. in.

(e) In the 36' barn with 16' and 12' members, the maximum bending moment created is 23,750 in. lbs. This moment produces a fiber stress of 2,713 #/sq.in.

(f) The maximum bending moment produced in the 36' barn with 16' and 14' members is 29,200 in. lbs. The maximum fiber stress is 2,410 #/sq.in.

(g) In the three rafter gambrel barns a moment of 39,460 in. lbs. was produced in the 36' barn with 14', 10', and 8' members. With a 2"x8" member the maximum fiber stress is 2,590 #/sq.in.

(h) In the 36' barn with 14', 12', and 8' rafter members, the maximum bending moment of 45,250 in. lbs. is produced. The moment produces a fiber stress of 2,970 #/sq.in. in a 2"x8" member.

(i) The maximum bending moment produced in the 40' barn with 14', 12', and 8' members is 45,150 in. lbs. This produces a fiber stress of 2,960 #/sq. in. in a 2" x 8" member.

(j) In the 40' barn with 14', 12', and 10' members, the maximum bending moment produced is 46, 100 in. lbs. The maximum fiber stress is 3,020 #/sq.in.

9. Horizontal shear and direct stress have no effect on rafter designs.

10. The end wind produces the greatest moment on the two rafter gambrel barn roofs.

11. The side wind produces the greatest moment on the three rafter gambrel barn roofs.

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